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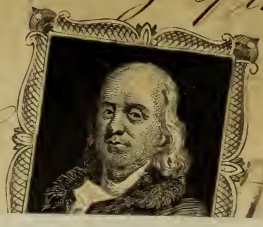

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METHOD OF CALCULATING

THE LOGARITHM

OF ANY

GIVEN NUMBER,

AND THE

NUMBER CORRESPONDING

TO ANY

GIVEN LOGARITHM.

DISCOVERED BY

OLIVER BYRNE,

Author and Inventor of the "Calculus of Form," a substitute for the Differential and Integral Calculus.

Author of "The New and Improved System of Logarithms;" "The Practical, Complete, and Correct Gager;" "A Treatise on Spherical Trigonometry;" "The Elements of Euclid, by Colors;" "How to Measure the Length of a Degree on the Earth's Surface, with the Assistance of Railroads;" etc., etc., etc.

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M.DCCC.XLIX.

Entered, according to Act of Congress, in the year 1849, by

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INTRODUCTION.

LOGARITHMS is as powerful an agent in calculation as steam is in mechanics; with this truth before us, it is strange that few know their proper use, or how they are computed. The most profound mathematician, or the most experienced calculator, has been hitherto unable, simply and directly, to compute in any reasonable time, the logarithm of a number taken at pleasure, or the number corresponding to a given logarithm to any required extent; for instance, if the logarithm of 365·25636516 be required to fifteen places of decimals,*

or the number corresponding to such an expression as $Y^{\frac{\varepsilon^9}{\pi^7}}$ to fifteen places of figures,† the labor necessary to produce the results without the help of tables or other extraneous aid is so great that such calculations are seldom if ever attempted; indeed, they may be said to be impracticable by any method previously proposed. But by the system which we shall presently explain and exemplify, the logarithm

* See example 28, page 24.

† $Y = 365{\cdot}24221685386$, the number of days in the *mean solar*, or *mean equinoctial year*, at the present time, namely, 1849.

$$\pi = 3{\cdot}141592653589794$$
$$\varepsilon = 2{\cdot}718281828459045$$

See example 59, page 47.

1726

of any given number, or the number answering to any given logarithm, may be directly calculated to any extent in less time than that occupied in extracting the square root. This scheme, too, may be acquired in a few minutes by any one who understands the ordinary rules of arithmetic; it depends chiefly on the numbers

1·371288574238542
237·5812087593121
3550·260181596591
46692·46832877758
576045·6934135527
6834720·776754357
78974890·31398144
895191589·8267839

the logarithms of which are composed of the same digits, or

log. 1·371288574238542 = 0·1371288574238542
log. 237·5812087593221 = 2·375812987593221
log. 3550·260181586591 = 3·550260181586581
log. 46692·45832877758 = 4·669246832877758
log. 576045·6934135527 = 5·760455934135527
log. 6834720·776754857 = 6·834720776754357
log. 78974890.31399144 = 7·897400031388144
log. 895171599·8267852 = 8·951915988257839

Since the days of Byrge, Briggs, and Vlacq, logarithmotechny, in a practical point of view, has received but little improvement; while logarithmic formulæ have been cultivated with great success, and advantageously employed to abridge many analytical inquiries in different parts of mathematics. However, it is also true, that some analysts have bestowed much time and labor in search of a simple and direct mode of calculating logarithms, and though wholly un-

successful, or very nearly so, as respects the ostensible object of the inquiry, they have been rewarded by the discovery of those interesting and momentous formulæ which constitute what is at present termed "the Theory of Logarithms."

It is also worthy of remark, that Briggs, Halley, Sharp, Vlacq, and others, who brought the doctrine of logarithms to perfection, were not averse to arithmetical calculations; but our modern mathematicians depend by far too much on purely algebraical expressions, foreign translations, and mere hocus pocus on the symbols of operation.

In an inquiry on logarithms, it is usual to put N = any given number, a = the base of any system, and M = the *modulus* of the system. Substituting $1 + n$ for N, &c., we have

$$\log.(1+n) = \mathrm{M}\ (n - \tfrac{1}{2}n^2 + \tfrac{1}{3}n^3 - \tfrac{1}{4}n^4 + \tfrac{1}{5}n^5 -,)\ \&c.,$$

for the fundamental expression, from which several other formulæ are derived, hitherto used in the computation of logarithms. But the above series is only useful when n is a very small fraction; while the majority of those deduced from it are only available in the process of determining logarithms from the combinations of others. The value of M, in the above series, cost Mr. Briggs 54 successive extractions of the square root, and 54 multiplications; and although many ingenious contrivances have been devised to abridge the labor of these extractions, the process is at best very tedious.

Lagrange converted the above series into

$$\log. m = r\,\mathrm{M}\left\{ (m^{\frac{1}{r}} = 1) - \frac{(m^{\frac{1}{r}} - 1)^2}{2} + \frac{(m^{\frac{1}{r}} - 1)^3}{3} - \&c. \right\}$$

by substituting $m^{\frac{1}{r}}$ for $1 + n$; r being entirely arbitrary. This formula can be rendered as convergent as we please, and therefore the

value of r can be so assumed, that the logarithm of any number, m, can be determined to a limited extent, by using only the first term of the series, viz., from the equation—

$$\log.\ m = r\,\mathrm{M}\,(m^{\frac{1}{r}} - 1).$$

This method, undoubtedly, is always applicable to the direct computation of a logarithm; yet it is the same in effect as that proposed by Briggs, and is equally laborious, on account of the great number of extractions generally required.

It is, perhaps, unnecessary to dwell at any great length on the difficulties attending the computation of logarithms by a direct process, independently of other logarithms; however, we cannot conclude these remarks without giving a remarkable expression, deduced by Professor Wallace, of Edinburgh. The form is this—

$$\log.\ x = \frac{b^{\frac{z}{m}} - 1}{x^{\frac{z}{n}} - 1} \cdot \frac{n\,(x^{\frac{1}{n}} - 1)}{m\,(b^{\frac{1}{m}} - 1)};$$

in which m and n are any numbers chosen at pleasure; z, always some value between 0 and 1; and b, the given base of the system. This expression leaves the base unrestricted, involves no infinite quantity, and is said by some to be "of great analytical elegance;" yet, it is purely algebraical, and as to its practical utility in the actual determination of a logarithm, it is of just as much use as other intelligible hieroglyphics.

Logarithms were invented by *Juste Byrge*, and not by Napier;*

* See "Biographie Universelle," "Encyclopædia Londinensis,' R. H. O'Byrne's "Representative History of Great Britain and Ireland," part 1, page 75; and "The Calculus of Form," 822, by Oliver Byrne, the author of the present work. According to Kepler, *Juste Byrge*, assistant astronomer to William Landgrave of Hesse, invented and pro-

mistakes of this kind are very common, especially in England. It is fully proved by Lagrange* and Laplace † that *Fermat*, and not Newton, invented the fluxiodal or differential calculus.‡ The binomeal theorem was not discovered by *Newton*, although it is engraved on his monument, in Westminster Abbey. Newton left no demonstration of this theorem; it was known for integral powers long before he was born; and *if he did find* that it holds when the indices are negative, the extension was not very great. William O'Neill, or as English writers term him, William Neal, was the first to rectify a curve of any sort; this curve was the semicutical parabola. Lord Bronnker, of Castle Lyons, Ireland, invented continued fractions, and was the first who quadrated the circle by means of series; yet these discoveries are ascribed to Wallis and Newton without the slightest foundation. The formula commonly, though erroneously, called *Cardan's Rule*, for cubic equations, was not invented by Cardan, but by Tartaglia, who communicated it to him under the strictest promise of secrecy. The theorem erroneously termed Maclaurin's was first given by James Stirling in his *Linæ Tertii Ordinis Newtoneanæ*, and ought to bear his name. The rule called General Roy's rule, was invented by Professor Dalby, and is but an application of *Albert Girard's* elegant rule.

Instances innumerable might be given to show that when a discovery, or an invention, is made in any part of the world, an English-

jected logarithms; he composed a table of sines for every two seconds of the quadrant long before Napier's time. Byrge was a Frenchman; M. Mansel especially mentions him as having invented the proportional compasses, which others have ascribed to *Galileo*. *Tycho Brahe*, also in his "Progymnasmata," vol. ii. mentions the works of *Byrge*. *Napier* was not the inventor of logarithms, he merely introduced them into England.

* Leçons sur le Calcul. des Fonctions, p. 321.

† System of the World, book v. chap. 3.

‡ Abbott's treatise on the Calculus of Variations, p. 2

man claims the honor of it immediately after. In our own time, Leverrier discovered a new planet, but the honor was thrust upon an Englishman by his government and scientific countrymen. Fitzgerald was the first to take out a patent for a steam-engine, but in no English history of that machine is either his name or his patent alluded to; it is added to the account of Watt, with many other things that do not belong to him. Every American is acquainted with the history of his own Fulton, and the attempts made to deprive him of the honor due to the originator of steam navigation; but Fitzgerald was an Irishman, and Fulton the son of one. Thomas Godfrey, of the city of Philadelphia, invented Hadley's quadrant—(see Blake's Biographical Dictionary). The discoveries of Dr. Matthew Young, and Dr. Richard Kirwan, of Dublin, Ireland, have become common property, and claimed by many pretenders. The honors due to Lucas Paccioli, known by the name of Lucas de Burgo, and his countryman, John de Sacrobosco, or John of Holywood, county Wicklow, Ireland, are distributed in the same unfair manner. The shameful treatment of Nicholas Mercator, who was born at Holstein in 1640, by the members of the Royal Society of England, deserves particular attention. The confusion becomes complete by introducing Gerard Mercator, an eminent geographer and mathematician, born in the low countries, in 1512, to whom we are indebted for the construction of those sea-charts, the honor of which is given to a man of the name of Wright. However, these charts are termed Mercator's charts. We are also indebted to Gerard Mercator for that part of navigation called Mercator's *Sailing*. G. Mercator died at Driesbourg in 1596. Between the two Mercators, Wright played the part of an English inventor well. This list might be continued without limit. About twenty years ago, I discovered this method of directly calculating logarithms. I could generally find the logarithm of any number in a minute or two without the use of books or tables. The importance of the discovery subjected me to all sorts of prying. Some asserted that I committed a table of logarithms to memory;

others attributed it to a peculiar mental property; and when societies and individuals failed to extract my secret, they never failed to traduce the inventor and the invention. Among the learned societies, the Royal Society of London played a very base part. When I have more space and time at my disposal, I will revert to this subject again.

The following tables will be found of use to the practical calculator.

TABLE OF PRIME NUMBERS TO 5000.

2	149	347	563	773	1019	1259	1499	1753
3	151	349	569	787	1021	1277	1511	1759
5	157	353	571	797	1031	1279	1523	1777
7	163	359	577	809	1033	1283	1531	1783
11	167	367	587	811	1039	1289	1543	1787
13	173	373	593	821	1049	1291	1549	1789
17	179	379	599	823	1051	1297	1553	1801
19	181	383	601	827	1061	1301	1559	1811
23	191	389	607	829	1063	1303	1567	1823
29	193	397	613	839	1069	1307	1571	1831
31	197	401	617	853	1087	1319	1579	1847
37	199	409	619	857	1091	1321	1583	1861
41	211	419	631	859	1093	1327	1597	1867
43	223	421	641	863	1097	1361	1601	1871
47	227	431	643	877	1103	1367	1607	1873
53	229	433	647	881	1109	1373	1609	1877
59	233	439	653	883	1117	1381	1613	1879
61	239	443	659	887	1123	1399	1619	1889
67	241	449	661	907	1129	1409	1621	1901
71	251	457	673	911	1151	1423	1627	1907
73	257	461	677	919	1153	1427	1637	1913
79	263	463	683	929	1163	1429	1657	1931
83	269	467	691	937	1171	1433	1663	1933
89	271	479	701	941	1181	1439	1667	1949
97	277	487	709	947	1187	1447	1669	1951
101	281	491	719	953	1193	1451	1693	1973
103	283	499	727	967	1201	1453	1697	1979
107	293	503	733	971	1213	1459	1699	1987
109	307	509	739	977	1217	1471	1709	1993
113	311	521	743	983	1223	1481	1721	1997
127	313	523	751	991	1229	1483	1723	1999
131	317	541	757	997	1231	1487	1733	2003
137	331	547	761	1009	1237	1489	1741	2011
139	337	557	769	1013	1249	1493	1747	2017

2027	2347	2683	2999	3343	3671	4003	4337	4673
2029	2351	2687	3001	3347	3673	4007	4339	4679
2039	2357	2689	3011	3359	3677	4013	4349	4691
2053	2371	2693	3019	3361	3691	4019	4357	4703
2063	2377	2699	3023	3371	3697	4021	4363	4721
2069	2381	2707	3037	3373	3701	4027	4371	4723
2081	2383	2711	3041	3389	3709	4049	4391	4729
2083	2389	2713	3049	3391	3719	4051	4397	4733
2087	2393	2719	3061	3407	3727	4057	4409	4751
2089	2399	2729	3067	3413	3733	4073	4421	4759
2099	2411	2731	3079	3433	3739	4079	4423	4783
2111	2417	2741	3083	3449	3761	4091	4441	4787
2113	2423	2749	3089	3457	3767	4093	4447	4789
2129	2437	2753	3109	3461	3769	4099	4451	4793
2131	2441	2767	3119	3463	3779	4111	4457	4799
2137	2447	2777	3121	3467	3793	4127	4463	4801
2141	2459	2789	3137	3469	3797	4129	4481	4813
2143	2467	2791	3163	3491	3803	4133	4483	4817
2153	2473	2797	3167	3499	3821	4139	4493	4831
2161	2477	2801	3169	3511	3823	4153	4507	4861
2179	2503	2803	3181	3517	3833	4157	4513	4871
2203	2521	2819	3187	3527	3847	4159	4517	4877
2207	2531	2833	3191	3529	3851	4177	4519	4889
2213	2539	2837	3203	3533	3853	4201	4523	4903
2221	2543	2843	3209	3539	3863	4211	4547	4909
2237	2549	2851	3217	3541	3877	4217	4549	4919
2239	2551	2857	3221	3547	3881	4219	4561	4931
2243	2557	2861	3229	3557	3889	4229	4567	4933
2251	2579	2879	3251	3559	3907	4231	4583	4937
2267	2591	2887	3253	3571	3911	4241	4591	4943
2269	2593	2897	3257	3581	3917	4243	4597	4951
2273	2609	2903	3259	3583	3919	4253	4603	4957
2281	2617	2909	3271	3593	3923	4259	4621	4967
2287	2621	2917	3299	3607	3929	4261	4637	4969
2293	2633	2927	3301	3613	3931	4271	4639	4973
2297	2647	2939	3307	3617	3943	4273	4643	4987
2309	2657	2953	3313	3623	3947	4283	4649	4993
2311	2659	2957	3319	3631	3967	4289	4651	4999
2333	2663	2963	3323	3637	3989	4297	4657	5003
2339	2671	2969	3329	3643	4001	4327	4663	5009
2341	2677	2971	3331	3659				

In all measurements, the decimal system of notation is incomparably the best; and it has always been found inconvenient in trigo-

nometry to have the arcs or angles measured in degrees, minutes, and seconds, while the linear quantities that represent them were decimally given. To remedy this defect I have constructed extensive tables, intended for publication. By this new arrangement, angles are measured in degrees, minutes, and decimal parts of a minute; thus—

$\frac{360^\circ}{7} = 51^\circ\ 57{\cdot}7142857,'$ the length of which on an arc to radius 1 will be ·8975979, &c. The length of an arc of one minute $= \frac{\pi}{180 \times 60} =$ ·00029088820866572158, &c.; radius = unity.

It is scarcely necessary to particularize the radius, unity being always understood when the contrary is not specified.

From having the length of an arc of one minute, the following useful table is easily constructed. By this table the length of an arc of any other number of degrees, minutes, &c., is readily found.

Tens of thousands.
Thousands.
Hundreds.
Tens.
Units.

1	0	0	0	0	0	0	0	0	2	9	0	8	8	8	2	0	8	6	7
2	0	0	0	0	0	0	0	0	5	8	1	7	7	6	4	1	7	3	3
3	0	0	0	0	0	0	0	0	8	7	2	6	6	4	6	2	5	9	9
4	0	0	0	0	0	0	0	1	1	6	3	5	5	2	8	3	4	6	6
5	0	0	0	0	0	0	0	1	4	5	4	4	4	1	0	4	3	3	3
6	0	0	0	0	0	0	0	1	7	4	5	3	2	9	2	5	1	9	9
7	0	0	0	0	0	0	0	2	0	3	6	2	1	7	4	6	0	6	6
8	0	0	0	0	0	0	0	2	3	2	7	1	0	5	6	6	9	3	3
9	0	0	0	0	0	0	0	2	6	1	7	9	9	3	8	7	7	9	9

Units.
Tenths.
Hundredths.
Thousandths.
Tenths of thousandths.
Hundredths of thousandths.

Required the length of an arc of 51° 25·714286′.

51° 25·714286′ = 3085·714286′.

For 3000·	.....	·8726646260
80·	.....	·0232710566
5·	.....	·0014544410
·7	.....	·0002036217
·01	.....	·0000029089
·004	.....	·0000011635
·0002	.....	·0000000582
·00008	.....	·0000000232
·000006	.....	·0000000017
3085·714286		·8975979008

Hence ·8975979008 is the length of an arc of 51° 25·714286′ radius = unity.

Find the number of degrees, minutes, &c., contained in an arc whose length is equal to that of the radius, by the foregoing table.

		Radius = 1·0000000000
3000·	the nearest in the table	·8726646260
		·1273353740
400·	.....	·1163552834
		·0109800906
30·	.....	·0087266463
		·0022534443
7·	.....	·0020362174
		·0002172269
·7	.....	·0002036217
		·0000136052
·04	.....	·0000116355

	·0000019697
·006	·0000017453
	·0000002244
·0007	·0000002036
	·0000000208
·00007	·0000000204
	·0000000004
·000001	·0000000003

$\therefore$ 3437·746771′ = 57° 17·746771′ = the degrees, minutes, &c., in an arc whose length is equal to radius.

What is the length of an arc of a circle of 180° ?

180° = 10800′

For	10000′· we find	2·9088820867
for	800· "	·2327105669
	10800·	3·1415926536

At page 53 we give the expression,

$$\text{Sin. } x = x - \frac{x^3}{1\cdot2\cdot3} + \frac{x^5}{1\cdot2\cdot3\cdot4\cdot5} - \frac{x^7}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7} + \text{&c.}$$

And, at page 56,

$$\text{cos. } x = 1 - \frac{x^2}{1\cdot2} + \frac{x^4}{1\cdot2\cdot3\cdot4} - \frac{x^6}{1\cdot2\cdot3.4\cdot5\cdot6} + \text{&c.};$$

x being the length of an arc to radius 1. To these may be added the following, taken from my work, the "Calculus of Form," a substitute for the Differential and Integral Calculus.

If x be an arc whose tangent is y, then

$$x = y - \frac{1}{3}\,y^3 + \frac{1}{5}\,y^5 - \frac{1}{7}\,y^7 + \text{&c.}$$

The arc $x + x = \frac{\pi}{2}$, then

$$x = \frac{1}{v} + \frac{1}{2\cdot3\cdot v^3} + \frac{1\cdot3}{2\cdot4\cdot5\cdot v^5} + \frac{1\cdot3\cdot5}{2\cdot4\cdot6\cdot7\cdot v^7} + \text{ \&c.}$$

And

$$x = \frac{\pi}{2} - \frac{1}{v} - \frac{1}{2\cdot3\cdot v^3} \text{ \&c.}$$

v being the secant of the arc x.

$$x = z + \frac{1\cdot z^3}{2\cdot3} + \frac{1\cdot3\, z^5}{2\cdot4\cdot5} + \frac{1\cdot3\cdot5\, z^7}{2\cdot4\cdot6\cdot7} + \text{ \&c.}$$

z being the sine of the arc x.

USEFUL FACTORS OFTEN USED IN CALCULATION, IN WHICH p REPRESENTS THE CIRCUMFERENCE OF A CIRCLE WHOSE DIAMETER IS 1.

$\frac{p}{4} = 4 \tan^{-1}\frac{1}{3} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99}$; and by obtaining the value of p we have

$p =$ 3·14159265358979323846264338327950288419716939
93751058209749445923078164062862089986280348
25342117067982148086513282306647093844609550
58223172535940812848473781392038633830215747
39960082593125912940183280651744 + &c., the circumference of a circle whose diameter is 1 = area of circle whose radius is 1.

$2p =$ 6·28318530717958647692528676655 9 the circumference of a circle whose radius is 1.

$4p =$ 12·56637061435917295385057353311 8 $= 2p \times 2$ the surface of a sphere whose radius is 1.

$36p =$ 113·09733552923255658465516179806 2 $= 4p \times 9$.

$\frac{1}{2}p =$ 1·57079632679489661923132169163 9 = semicircle whose diameter is 1 = quadrant whose radius is 1.

$\frac{1}{4}\,p = 0{\cdot}78539816339744830961566084581\,9 = \frac{1}{2}$ of $\frac{p}{2}$ = quadrant whose diameter is 1 = area of a circle whose diameter is 1.

$\frac{1}{6}\,p = 0{\cdot}52359877559829887307710723054\,6 = \frac{2}{3}$ of $\frac{p}{4}$ = arc of 60° to diameter 1 = solidity of a sphere whose diameter is 1.

$\frac{1}{8}\,p = 0{\cdot}39269908169872415480783042290\,9 = \frac{3}{4}$ of $\frac{p}{6}$ = arc of 45° to diameter 1.

$\frac{1}{12}\,p = 0{\cdot}26179938779914943653855361527\,3 = \frac{2}{3}$ of $\frac{p}{8}$ = arc of 30° to diameter 1.

$\frac{1}{24}\,p = 0{\cdot}13089969389957471826927680763\,6 = \frac{1}{2}$ of $\frac{1}{12}\,p$ = arc of 15° to diameter 1.

$\frac{1}{180}\,p = 0{\cdot}01745329251994329576923690768\,4 = \frac{2}{15}$ of $\frac{1}{24}\,p$ = arc of 1° to radius 1.

$\frac{1}{360}\,p = 0{\cdot}00872664625997164788461845384\,2 = \frac{1}{2}$ of $\frac{1}{180}\,p$ = arc of 30′ to radius 1.

$\frac{1}{10800}\,p = 0{\cdot}00029088820866572159615394846\,14 = \frac{1}{30}$ of $\frac{1}{360}\,p$ = arc of 1′ to radius 1.

$\frac{1}{6480000}\,p = 0{\cdot}00000484813681109535993589914\,102 = \frac{1}{60}$ of $\frac{1}{10800}\,p$ = arc of 1″ to radius 1.

$\frac{2}{3}\,p = 2{\cdot}09439510239319549230842892218\,6 = \frac{1}{6}\,p \times 4$ = arc of 120° to radius 1.

$\frac{4}{3}\,p = 4{\cdot}18879020478639098461685784437\,2 = \frac{2}{3}\,p \times 2$ = arc of 240° to radius 1 = solidity of a sphere whose radius is 1.

$r^\circ = 57^\circ{\cdot}2957795131 = 57^\circ\,.\,17'\,.\,44''{\cdot}80624$ the degrees in an arc = radius.

$r' = 3437'{\cdot}7467707849 = 3437'\,.\,44''{\cdot}80624$ the minutes in an arc = radius.

$r'' = 206264''{\cdot}8062470963$ the number of seconds in an arc = radius.

$\sqrt{2} = 1{\cdot}41421356237309504880168872420 97$ the diameter of a circle of which side of inscribed square is 1.

$\sqrt{\frac{1}{2}} = 0{\cdot}70710678118654752440084436210 48$ the reciprocal of $\sqrt{2}$ = the side of a square inscribed in a circle whose diameter is 1.

$p\sqrt{2} = 4{\cdot}44288293815826624701588099006 05.$

$p\sqrt{\frac{1}{2}} = 2{\cdot}22144146907918312350794049503 02 = \frac{1}{2}$ of $p\sqrt{2}$.

$p^2 = 9{\cdot}86960440108935861883449099987 6.$

$6\,p^2 = 59{\cdot}21762640653615171300694599925 6.$

$\frac{1}{p} = 0{\cdot}31830988618379067153776752674 50$ the reciprocal of p.

$\frac{2}{p} = 0{\cdot}63661977236758134307553505349 0 = \frac{1}{p} \times 2.$

$\frac{4}{p} = 1{\cdot}27323954473516268615107010698 0 = \frac{2}{p} \times 2.$

$\frac{6}{p} = 1{\cdot}90985931710274402922660536047 0 = \frac{3}{2}$ of $\frac{4}{p}$.

$\frac{360}{p} = 114{\cdot}59155902616464175359630962820 0.$

$\frac{1}{2p} = 0{\cdot}15915494309189533576888376337 25 = \frac{1}{2}$ of $\frac{1}{p}$.

$\frac{1}{4p} = 0{\cdot}07957747154594766788444188168 62 = \frac{1}{2}$ of $\frac{1}{2p}$ = area of a circle whose circumference is 1.

$\frac{1}{6p} = 0{\cdot}05305164769729844525629458779 06 = \frac{2}{3}$ of $\frac{1}{4p}$.

$\frac{1}{p}\sqrt{2} = 0{\cdot}45015815807855303477759955 02.$

$\frac{1}{p}\sqrt{\frac{1}{2}} = 0.22507907903927651738879977 51 = \frac{1}{2}$ of $\frac{1}{p}\sqrt{2}$.

$\frac{1}{p^2} = 0{\cdot}10132118364233777144387946320 9.$

$\frac{1}{2p^2} = 0{\cdot}05066059182116888572193973160 4.$

$\frac{1}{6p^2} = 0{\cdot}016886863940389628573979910534.$

$\sqrt{p} = 1{\cdot}772453850905516027298167483341$ is the side of a square = in surface to a sphere whose diameter is 1.

$2\sqrt{p} = 3{\cdot}544907701811032054596334966682$ the circumference of a circle whose area is 1.

$\frac{1}{2}\sqrt{p} = 0{\cdot}886226925452758013649083741670 = \frac{1}{4}$ of $2\sqrt{p}$ = the side of a square = circle whose diameter is 1.

$\frac{1}{8}\sqrt{p} = 0{\cdot}221556731363189503412270935418 = \frac{1}{4}$ of $\frac{1}{2}\sqrt{p}$.

$\sqrt{\frac{p}{2}} = 1{\cdot}253314137315500251207882642402 = \frac{1}{2}\sqrt{2p}.$

$\sqrt{\frac{1}{p}} = 0{\cdot}564189583547756286948079451560$ the reciprocal of $\sqrt{p}$ = the diameter of a sphere whose surface is 1.

$2\sqrt{\frac{1}{p}} = 1{\cdot}128379167095512573896158903120$ the diameter of a circle = square whose side is 1.

$\frac{1}{2}\sqrt{\frac{1}{p}} = 0{\cdot}282094791773878143474039725780 = \frac{1}{4}$ of $2\sqrt{\frac{1}{p}}.$

$\frac{1}{6}\sqrt{\frac{1}{p}} = 0{\cdot}094031597257959381158013241926 = \frac{1}{3}$ of $\frac{1}{2}\sqrt{\frac{1}{p}}.$

$\frac{1}{8}\sqrt{\frac{1}{p}} = 0{\cdot}070523697943469535868509931445 = \frac{3}{4}$ of $\frac{1}{6}\sqrt{\frac{1}{p}}.$

$\sqrt{\frac{2}{p}} = 0{\cdot}797884560802865355879892119868 = \frac{1}{p}\sqrt{2p}.$

$\sqrt[3]{36p} = 4{\cdot}83597586204,$ &c.

$\sqrt[3]{\frac{p}{6}} = 0{\cdot}805995977007,$ &c. = the side of a cube = to a sphere whose diameter is 1.

$\sqrt[3]{6p^2} = 3{\cdot}89777707,$ &c. = the periphery of a sphere whose solid content is 1.

$\sqrt[3]{\frac{6}{p}} = 1{\cdot}2407009819,$ &c. = the diameter of a sphere whose solid content is 1.

$\surd$ 277·274, otherwise $\surd$ 277·273843570 = 16·651541777565, &c.

$\sqrt{\frac{277\cdot274}{\cdot785398,\ \&c.}}$, otherwise $\sqrt{\frac{277\cdot273843570}{\cdot7853981633974483,\ \&c.}}$ = 18·789252841825, &c.

$\frac{1}{277\cdot274}$ = 0·00360654082243556914820718855 7, &c.

$\surd$ 231 = 15·1986841535706636.

$\surd$ 282 = 16·7928556237466, &c.

$\sqrt{\frac{282}{\cdot785398,\ \&c.}}$ = 18·948708, &c.

$\surd$ 10152 = 100·75713374.

Common log. of 10 = 1.

Hyperbolic log. of 10 = 2·302585092994045684017991454684=k.

Reciprocal of ditto = 0·434294481903251827651128918916 = $\frac{1}{k}$ = com. log. of 2·718281828459045235360287471352.

Hyperbolic log. of 2·718281828459045235360287471352 = 1.

Common log. of p = 0·49714987269413385435 = m.

Hyperbolic log. of p = 1·144729885849400174 14 = km.

Common log. of r° = 1·758122632409172215 45.

Common log. of r' = 3·536273882792815847 96.

Common log. of r'' = 5·314425133176459480 47.

Note. It has been found by experiment, that, at the temperature of 62° of Fahrenheit, 1 cubic inch of distilled water weighs 252·722 grains troy in a vacuum, or 252·456 grains in air; and that at the maximum of density, that is at 39°, it weighs in a vacuum 253 grains troy. Hence, at the temperature of 62°, 1 cubic foot of distilled water weighs 998·17969 avoirdupois ounces in a vacuum, or 997·129069 avoirdupois ounces in air; and at the temperature of 39°, it weighs 999·2777 avoirdupois ounces in a vacuum.

The troy ounce measure of water is equal to 1·901307$\frac{3}{7}$ cubic inches,
The avoirdupois ounce measure of water is equal to 1·7329$\frac{5}{8}$ cubic inches,
} at the temperature of 62° of Fahrenheit.

A cubic foot, or 1728 cubic inches, of air weighs 528·367 grains troy, or 1·207696 ounces avoirdupois.

The French *Metre* = 3·2808992 English *feet* linear measure = 39·3707904 *inches.*

For Multiples the following Greek words are used.	*For Divisors the following Latin words are used.*
Deca for ..... 10 times.	*Deci* for the 10*th* part.
Hecto " ..... 100 times.	*Centi* " 100*th* part.
Kilo " ..... 1000 times.	*Milli* " 1000*th* part.
Myria " ..... 10000 times.	Thus a *Kilometre* = 1000 metres.
	$Millimetre = \frac{\text{metre}}{1000}$

The square of *Deca Metre*, called the *Are*, is the element of land measure in France which = 1076·42996 *square feet* English.

The *Stere* is a cubic metre = 35·316582 *cubic feet*, English.

The *Litre* for liquid measure is a cubic decimetre = 1·76077 *imperial pints*, English, at the temperature of melting ice; a *litre* of distilled water weighs 15434 *grains troy.*

The unit of weight is the *gramme;* it is the weight of a cubic centimetre of distilled water, or of a millilitre, and therefore equal to 15·434 *grains troy.*

LOGARITHMS.

1. When

$$a^x = b,$$

that is, when a in the x power is equal to b, x is termed the logarithm of b to the base a.

If a be 10, the base of the common system of logarithms, x is the common logarithm of b.

Since $10^2 = 100$, $10^3 = 1000$, $10^4 = 10000$, &c., we have log. of $100 = 2\cdot$; log. of $1000 = 3\cdot$; log. of $10000 = 4\cdot$, &c.

Again: $10^{-2} = \frac{1}{10^2}$; $10^{-3} = \frac{1}{10^3}$; $10^{-4} = \frac{1}{10^4}$; &c.

Hence log. 10^{-2} = log. $\frac{1}{10^2}$ = log. $\cdot 01 = -2$ or $\bar{2}$;

And log. 10^{-3} = log. $\frac{1}{10^3}$ = log. $\cdot 001 = -3$, or $\bar{3}$, and so on.

Consequently, of the two series,

$\cdot\circ\chi$ -4; -3; -2; -1; 0; 1; 2; 3; 4; &c.

$\cdot\circ\chi$ $\cdot 0001$; $\cdot 001$; $\cdot 01$; $\cdot 1$; $1\cdot$; 10; 100; 1000; 10000; &c.;

the first, being in an arithmetical progression, gives the logs. of the corresponding numbers in the second series, which is in a geometrical progression.

2. It may be necessary to show, that

$$10^0 = 1, 10^{-1} = \cdot 1,\ 10^{-2} = \cdot 01,\ 10^{-3} = \cdot 001, \text{ &c.}$$

$a^2 = \frac{a^5}{a^3} = \frac{a^2}{1}$, by the same rule $a^{-2} = \frac{a^3}{a^5} = \frac{1}{a^2}$; and $a^0 = \frac{a^3}{a^3} = 1$.

$\therefore 10^0 = 1, 10^{-1} = \cdot 1, 10^{-2} = \cdot 01,\ 10^{-3} = \cdot 001$, &c.

3. It will be shown hereafter (19), that the logarithm of 542470· is 5·7343757, or as it is usually written,

$$\log.\ 542470\cdot = 5\cdot7343757$$
$$\therefore\ 10^{5.7343757} = 542470.$$

4. From inspecting the arithmetical and geometrical series just given, it is clear that all numbers between 10 and 100 must have a logarithm commencing with 1; all numbers between 100 and 1000 will have a logarithm commencing with 2; all numbers between 1000 and 10000 will have a logarithm beginning with 4; and so on.

Hence,

$$\log.\ 542470\cdot = 5\cdot7343757$$
$$\log.\ 54247\cdot0 = 4\cdot7343757$$
$$\log.\ 5424\cdot70 = 3\cdot7343757$$
$$\log.\ 542\cdot470 = 2\cdot7343757$$
$$\log.\ 54\cdot2470 = 1\cdot7343757$$
$$\log.\ 5\cdot42470 = 0\cdot7343757$$
$$\log.\ \cdot542470 = \bar{1}\cdot7343757$$

which signifies *minus* 1, *plus* ·7343757, and may be written,

$$-1 + (\cdot7343757) = -(\cdot2656243).$$
$$\log.\ \cdot054247 = \bar{2}\cdot7343757$$
$$\log.\ \cdot0054247 = \bar{3}\cdot7343757$$
$$\log.\ \cdot00054247 = \bar{4}\cdot7343757$$
$$\&c.\quad \&c.$$

5. These properties and conventional arrangements are so well known, to say more respecting them would be superfluous.

The succeeding numbers possess a particular property which is worth being remembered.

$$1\cdot371288574238542 = 10^{\cdot1371288574238542}$$
$$10\cdot00000000000000 = 10^{1.000000000000000}$$
$$237\cdot5812087593221 = 10^{2.375812087593221}$$
$$3550\cdot260181586591 = 10^{3.550260181586591}$$
$$46692\cdot46832877758 = 10^{4.669246832877758}$$
$$576045\cdot6934135527 = 10^{5.760456934135527}$$
$$6834720\cdot776754357 = 10^{6.834720776754357}$$
$$78974890\cdot31398144 = 10^{7.897489031398144}$$
$$895191599\cdot8267839 = 10^{8.951915998267839}$$
$$9999999999\cdot999999 = 10^{9.999999999999999}$$

6. Therefore,

$$\log.\ 1\cdot371288574238542 = 0\cdot1371288574238542$$
$$\log.\ 10\cdot00000000000000 = 1\cdot000000000000000$$
$$\log.\ 237\cdot5812087593221 = 2\cdot375812087593221$$
$$\log.\ 3550\cdot260181586591 = 3\cdot550260181586591$$
$$\log.\ 46692\cdot46832877758 = 4\cdot669246832877758$$

log. 576045·6934135527 = 5·760456934135527
log. 6834720·776754357 = 6·834720776754357
log. 78974890·31398144 = 7·897489031398144
log. 895191599·8267852 = 8·951915998267839
log. 9999999999·999999 = 9·999999999999999

7. In these numbers, if the decimal points be changed, it is evident the logarithms corresponding can also be set down without any calculation whatever.

Thus, the log. of 137·1288574238542 = 2·1371288574238542;
the log. of 35·50260181586591 = 1·550260181586591;
log. ·002375812087593221 = $\bar{3}$·375812087593221;
log. ·0008951915998267852 = $\bar{4}$·951915998267852;
And so on in similar cases, since the change of the decimal point in a number can only affect the whole number of its logarithm.

8. These numbers whose logarithms are made up of the same digits will be found extremely useful hereafter. We shall next give a simple method of multiplying any number by any power of 11, 101, 1001, 10001, 100001, &c.

This multiplication is performed by the aid of coefficients of a binomial raised to the proposed power.

$(x+y)^1 = x+y$, the coefficients are 1, 1.
$(x+y)^2 = x^2+2xy+y^2$, the coefficients are 1, 2, 1.
$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$, the coefficients are 1, 3, 3, 1.
The coefficients of $(x+y)^4$ are 1, 4, 6, 4, 1.
" " $(x+y)^5$ " 1, 5, 10, 10, 5, 1.
" " $(x+y)^6$ " 1, 6, 15, 20, 15, 6, 1.
" " $(x+y)^7$ " 1, 7, 21, 35, 35, 21, 7, 1.
" " $(x+y)^8$ " 1, 8, 28, 56, 70, 56, 28, 8, 1.
" " $(x+y)^9$ " 1, 9, 36, 84, 126, 126, 84, 36, 9, 1.

9. Let it be required to multiply 54247 by $(101)^6$.

The number must be divided into periods of two figures when the multiplier is 101; into periods of three figures when the multiplier is 1001; into periods of four figures when the multiplier is 10001; and so on.

e	d	c	b	a			
	54	24	70	00	00		1
	3	25	48	20	00	a	6
		8	13	70	50	b	15
			10	84	94	c	20
				8	14	d	15
					3	e	6

$(54247) \times (101)^6 =$ 57 58 42 83 61, true to 10 places of figures.

This operation is readily understood, since the multipliers for the 6th power are 1, 6, 15, 20, 15, 6, 1; we begin at a, a period in ad-

vance, and multiply by 6; then we commence at b, two periods in advance, and multiply by 15; at c, three periods in advance, and multiply by 20; at d, four periods in advance (counting from the right to the left), and multiply by 15; the period, e, should be multiplied by 6, but, as it is blank, we only set down the 3 carried from multiplying d, or its first figure by 6.

10. As it is extremely easy to operate with 1, 5, 10, 10, 5, 1, the multipliers for the 5th power, it may be more convenient first to multiply the given number by $(101)^5$, and then by $(101)^1$; because, to multiply any number by 5, we have only to affix a cipher (or suppose it affixed) and to take the half of the result.

The above example, if worked in the manner just described, will stand as follows:

d	c	b	a		
54	24	70	00	00	...1
2	71	23	50	00	...5..a
	5	42	47	00	..10..b
		5	42	47	..10..c
			2	71	...5..d
				1	...1

$$(44247) \times (101)^5 = \begin{array}{|c|c|c|c|c|} \hline 57 & 01 & 41 & 42 & 19 \\ \hline & 57 & 01 & 41 & 42 \\ \hline \end{array}$$

$$57\ 58\ 42\ 83\ 61 = (542471)^6 \times (101)^6.$$

11. The truth of this is readily shown by common multiplication, but the process is cumbersome. However, for the sake of comparison, we shall in this instance multiply 54247 by (101) raised to the 6th power.

$$\begin{array}{r} 101 \\ 101 \\ \hline 101 \\ 1010 \\ \hline 10201 \\ 101 \\ \hline 10201 \\ 102010 \\ \hline 1030301 \\ 101 \\ \hline 1030301 \\ 10303010 \\ \hline 104060301 \\ 101 \\ \hline 104060401 \\ 1040604010 \\ \hline 10510100501 \end{array} \begin{array}{l} \\ \\ \\ \\ = (101)^2. \\ \\ \\ \\ = (101)^3. \\ \\ \\ \\ = (101)^4. \\ \\ \\ \\ = (101)^5. \end{array}$$

$$\begin{array}{r} 10510100501 = (101)^5. \\ 101 \\ \hline 10510100501 \\ 105101005010 \\ \hline 1061520150601 = (101)^6. \\ 54247 \\ \hline 7430641054207 \\ 4246080602404 \\ 2123040301202 \\ 4246080602404 \\ 5307600753005 \\ \hline 575842836096652447 \text{ the required product,} \end{array}$$

which shows that the former process gives the result true to 10 places of figures, of which we shall add another example.

12. Multiply 34567812 by $(1001)^8$, so that the result may be true to twelve places of figures.

c	b	a		
	3456	7812	0000	...1
	2	7654	2496	...8..a
		9	6790	..28..b
			19	..56..c

3459 5475 9305 the required product.

The remaining multipliers, 70, 56, 28, 8, 1, are not necessary in obtaining the first twelve figures of the product of 34567812 by 10001 in the 8th power.

13. As 28 and 56 are large multipliers, the work may stand thus:

c	b	a			
	3456	7812	0000	. . . 1	
	2	7654	2496	. a . 8	
		6	9136	. b . 20	28
		2	7654	. b . 8	
			17	. c . 50	56
			2	. c . 6	

Result, = 345954759305 the same as before.

14. Perhaps this product might be obtained with greater ease by first multiplying 34567812 by $(10001)^5$, and the product by $(10001)^3$; the operation will stand thus:

$$\begin{array}{r} 345678120000 \ldots 1 \\ 172839060 \ldots 5 \\ 34568 \ldots 10 \\ 3 \ldots 10 \\ \hline 345859093631 = 34567812 \times (10001)^5. \end{array}$$

$345850093631 = 34567812 \times (10001)^5.$
$103755298 \ldots 3$
$10376 \ldots 3$
345954759305 = twelve places of the product of 34567812 by $(10001)^5 \times (10001)^3 = (34567812) \times (10001)^8$.

Although these methods are extremely simple, yet cases will occur, when one of them will have the preference.

15. Our next object is to determine the logarithms 1·1; 1·01; 1·001; 1·0001; 1·00001; &c.

It is well known that

$\log.(1+n) = \mathrm{M}\,(n - \frac{1}{2}n^2 + \frac{1}{3}n^3 - \frac{1}{4}n^4 + \frac{1}{5}n^5 - \frac{1}{6}n^6 + \text{ &c.})$

M being the modulus, $=$·43294481903261827651128 9, &c.

It is evident that when n is $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1006}$, $\frac{1}{10000}$, &c., the calculation becomes very simple.

$\mathrm{M} =$ ·4342944819032518
$\frac{1}{2}\mathrm{M} =$ ·2171472409516259
$\frac{1}{3}\mathrm{M} =$ ·1447648273010839
$\frac{1}{4}\mathrm{M} =$ ·1085736204758130
$\frac{1}{5}\mathrm{M} =$ ·0868588963806504
$\frac{1}{6}\mathrm{M} =$ ·0723824136505420
$\frac{1}{7}\mathrm{M} =$ ·0720420788433217
$\frac{1}{8}\mathrm{M} =$ ·0542868102379065
$\frac{1}{9}\mathrm{M} =$ ·0482549424336946
$\frac{1}{10}\mathrm{M} =$ ·0434294481903252

&c. &c., are constants employed to determine the logarithms of 11, 101, 1001, 100001, &c.

16. *To compute the log. of* 1·001. In this case $n = \frac{1}{1000}$.

$$+\frac{\mathrm{M}}{1000} = \text{·0004342944819033 positive}$$
$$-\frac{\frac{1}{2}\mathrm{M}}{(1000)^2} = \underline{\text{·0000002171472410 negative}}$$
$$\text{·0004340773346623}$$
$$+\frac{\frac{1}{3}\mathrm{M}}{(1000)^3} = \underline{\text{·0000000001447648 positive}}$$
$$\text{·0004340774794271}$$
$$-\frac{\frac{1}{4}\mathrm{M}}{(1000)^4} = \underline{\text{·0000000000001086 negative}}$$
$$\text{·0004340774793185}$$
$$+\frac{\frac{1}{5}\mathrm{M}}{(1000)^5} = \underline{\text{·0000000000000001 positive}}$$
$$\text{·0004340774793186} = \text{the log. of } 1{\cdot}001;$$

true to sixteen places.

17. It is almost unnecessary to remark, that, instead of adding and subtracting alternately, as above, the positive and negative terms may be summed separately, which will render the operation more concise.

POSITIVE TERMS.	NEGATIVE TERMS.
·0004342944819033	·0000002171472410
1447648	1086
1	·0000002171473496
+ ·0004342945266682	
— 000000217473496	

·0004340774793186 = log. 1·001.

18. In a similar manner the succeeding logarithms may be obtained to almost any degree of accuracy.

Log. 1·1	= ·041392685158225	&c. which we call		A
" 1·01	= ·004321373782643	"	"	B
" 1·001	= ·000434077479319	"	"	C
" 1·0001	= ·000043427276863	"	"	D
" 1·00001	= ·000004342923104	"	"	E
" 1·000001	= ·000000434294265	"	"	F
" 1·0000001	= ·000000043429447	"	"	G
" 1·00000001	= ·000000004342945	"	"	H
" 1·000000001	= ·000000000434295	"	"	I
" 1·0000000001	= ·000000000043430	"	"	J
" 1·00000000001	= ·000000000004343	"	"	K
" 1·000000000001	= ·000000000000434	"	"	L
" 1·0000000000001	= ·000000000000043	"	"	M
" 1·00000000000001	= ·000000000000004	"	"	N
&c.	&c.			&c.

Without further formality or paraphernalia, for it is presumed that such is not necessary, we shall commence operating, as the method can be acquired with ease, and put in a clearer point of view by proper examples.

19. Required the logarithm of 542470 (3), to seven places of decimals.

5 4 | 2 4 | 7 0 | . .		(9.)
3 | 2 5 | 4 8 | 2 0		
8 | 1 3 | 7 1		
1 0 | 8 5		
8		
5 7 5 8 | 4 2 8 4	= 6B = ·02592824	See (18)
1 | 7 2 7 5		
3		
Take 5 7 6 0 1 5 6 2	= 3D = ·00013028	
From 5 7 6 0 4 5 6 9		
576) · · · · 3 | 0 0 7		
2 | 8 8 0	= 5E = ·00002171	
1 | 2 7		
1 | 1 5	= 2F = ·00000087	

$$\begin{array}{r} 1\,|\,2 \\ 1\,|\,2 \end{array} = 2\,G = \cdot 00000009$$

$$\cdot 02608119 \text{ Take}$$

$$5\cdot 76045693 \text{ From} \quad (6)$$

Hence we have log. 542470 = 5·73437574, which is correct to seven decimal places.

20. 6 B is written to represent 6 times the log. of 1·01 (18).

The nearest number to 542470, whose log. is composed of the same digits as itself, being 576045·6934, &c. (6), our object was to raise 542470· to 576045·69 by multiplying 542470· by some power or powers of 1·1, 1·01, 1·001, 1·0001, &c.

It is here necessary to remark, that A is not employed, because the given number multiplied by 1·1, would exceed 576045·69; for a like reason C is omitted.

Again, when half the figures coincide the process may be performed (as above) by common division; the part which coincides becoming the divisor; thus, in finding 5 E, 576 is divided into 3007, it goes 5 times, the E showing that there are five figures in each period at this step. For A, there is but one figure in each period; for B, there are two figures; for C, there are three figures in each period, and so on.

21. Let it be required to calculate the logarithm of 2785·9, true to seven places of decimals.

It will be found more convenient, in this instance, to bring the given number to 3550·26018, the log. of which is 3·55026908 (6).

$$\begin{array}{r} 2\,|\,7\,|\,8\,|\,5\,|\,9\,|\,0\,|\,0\,|\,0 \\ \quad 5\,|\,5\,|\,7\,|\,1\,|\,8\,|\,0\,|\,0 \\ \quad\quad 2\,|\,7\,|\,8\,|\,5\,|\,9\,|\,0 \end{array}$$

$$\begin{array}{r} 3\,3\,|\,7\,0\,|\,9\,3\,|\,9\,0 \\ \quad 1\,|\,6\,8\,|\,5\,4\,|\,7\,0 \\ \quad\quad\quad 3\,|\,3\,7\,|\,0\,9 \\ \quad\quad\quad\quad\quad 3\,|\,3\,7 \\ \quad\quad\quad\quad\quad\quad\quad 2 \end{array} = 2\,A = \cdot 08278537 \quad (18).$$

$$\begin{array}{r} 3\,5\,4\,|\,2\,8\,9\,|\,0\,8 \\ \quad\quad\quad 7\,0\,8\,|\,5\,8 \\ \quad\quad\quad\quad\quad\quad 3\,5 \end{array} = 5\,B = \cdot 02160687$$

$$\begin{array}{rr} \text{Take} & 3\,5\,4\,9\,|\,9\,8\,0\,1 \\ \text{From} & 3\,5\,5\,0\,|\,2\,6\,0\,2 \end{array} = 2\,C = \cdot 00086815$$

$$\begin{array}{r} 355) \quad \cdot\,\cdot\,\cdot\,\cdot\,2\,|\,8\,0\,1 \\ \quad\quad\quad\quad 2\,|\,4\,8\,5 \end{array} = 7\,E = \cdot 00003040$$

$$\begin{array}{r} \quad\quad\quad\quad\quad 3\,|\,1\,6 \\ \quad\quad\quad\quad 2\,|\,8\,4 \end{array} = 8\,F = \cdot 00000347$$

$$\begin{array}{r} \quad\quad\quad\quad\quad\quad 3\,|\,2 \\ \quad\quad\quad\quad\quad 3\,|\,2 \end{array} = 9\,G = \cdot 00000039$$

$$\cdot 10529465$$

Take ·10529465
From 3·55026018
log. 2785·9 = 3·44496553.

22. At the Observatory at Paris, $g = 9{\cdot}80896$ metres, the second being the unit of time, what is the logarithm of 9·80896.

In this example we shall bring 9·80896 to 9·99999, &c. (6).

9 8|0 8|9 6|0 0|0 0
9 8|0 8|9 6|0 0
9 9 0|7 0 4|9 6 0|0 = 1 B = ·0043213738
8|9 1 6|3 4 4|6
3 5|6 6 5|4
8 3|2
9 9 9 6|5 7 0 5|3 2 = 9 C = ·0039066973
2|9 9 8 9|7 2
3|0 0
9 9 9 9 5|6 9 8 0 4 = 3 D = ·0001302818
3|9 9 9 8 3
6
Take 9 9 9 9 9 6 9 7 9 3 = 4 E = ·0000173717
From 1 0 0 0 0 0 0 0 0 0 0
· · · · · 3 0 2 0 7

From which we have..... 3 F = ·0000013029
2 H = ·0000000087
7 J = ·0000000003
Take ·0083770365
From ·0000000000
Log. 9·80896 = ·9916229635

23. As before observed, 9 C might have been obtained in the following manner:

8 9 0|7 0 4|9 6 0|0 = 1 B as above
4|9 5 3|5 2 4|8
9|9 0 7|0
9|9
5 times 9 9 5|6 6 8|4 0 1|7
3|9 8 2|6 7 3|6
5|9 7 3|9
4|0
4 times 9 9 9 6 5 7 0 5 3 2 = 9 C.

24. A French metre is equal 3·2808992 English feet, required the log. of 3·2808992.

e	d	c	b	a	
	32	80	89	92	00..once
	2	29	66	29	44.. 7 times from *a*
		6	88	98	88..21 " " *b*
			11	48	31..35 " " *c*
				11	48..35 " " *d*
					7..21 " " *e*

35 17 56 80 18 = B 7.

25. The manner in which B 7 is obtained is worthy of remark; the multipliers being 1, 7, 21, 35, 35, 21, 7, 1, (8,) when 7 times the first line (commencing with the period marked *a*) is obtained, 21 times the same line (commencing with the period marked *b*) is determined by multiplying the 2nd line by 3. If the 2nd line be again multiplied by 5, we have the 4th line for the multiplier 35; but to multiply by 5, we have only to take the half the product produced by multiplying by 7, advancing the result one figure to the right. Hence, to find the result for 35 is almost as easy as to find the result for 5.

But the object in this case being to bring the proposed number to 35502601815, the process must be continued.

	c	b	a	
1	351	756	801	8 = B 7 as above
9	3	165	811	2
36		12	663	2
84			29	6

354 935 305 8 = C 9.

26. The 2nd (or 9) line is produced by beginning at *a*, but the multiplication may be performed by subtracting 3517568 from 35175680; the 36 line is produced by beginning at *b*, observing to carry from the preceding figure, making the usual allowance when the number is followed by 5, 6, 7, 8, or 9. The 36 line may be produced by multiplying the 9 line by 4, beginning one period more to the left. To multiply by 84 is not apparently so convenient, for $84 \times 352 = 29|568$; and as only one figure of the period 568 is required, when the proper allowance is made, the result becomes 29|6.

But, since 84 is equal to $36 \times 2\frac{1}{3}$, we have only to multiply the 36 line by 2, and add $\frac{1}{3}$ of it; with such management the work will stand thus:—

351	756	801	8	= B 7 as before
3	165	811	2	= 9 times
	12	663	2	= 36 times
		24	3	= 72 times } = 84 times
		4	2	= 12 times }

354 935 305 8 = C 9.

This amounts to very little more than adding the above numbers together.

27. Many other contractions will suggest themselves, when the multipliers are large: thus to multiply any number 57837 by 9, as alluded to above (26.), is easily effected, by the following well known process:—Subtract the first figure to the right from 10, the second from the first, the third from the second, and so on.

$$\text{Thus } 57837 \times 9 = \left\{ \begin{array}{l} 578370\,..\text{ten times} \\ \underline{\;\;57837}\,..\text{once} \\ 520533\,..\text{nine times.} \end{array} \right.$$

Such simple observations are to be found in every book on mental arithmetic, and therefore require but little attention here.

The whole work of the previous example will stand thus:—

```
          3 2|8 0|8 9|9 2|0 0
            2|2 9|6 6|2 9|4 4
                6|8 8|9 8|8 8
                  1 1|4 8|3 1
                      1 1|4 8 + 7
B 7 = 3 5 1|7 5 6|8 0 1|8 = ·0302496165 = 7 B
            3|1 6 5|8 1 1|2
                1 2|6 6 3|2
                      2 9|6
C 9 = 3 5 4 9|3 5 3 0|5 8 = ·0039066973 = 9 C
              |7 0 9 8|7 1
                      |3 5
D 2 = 3 5 5 0 0|6 2 9 6 4 = ·0000868546 = 2 D
                1|7 7 5 0 3
                          4
Take E 5 = 3 5 5 0 2 4 0 4 7 1 = ·0000217146 = 5 E
From       3 5 5 0 2 6 0 1 8 2 (6)
3550 ) . . . . . 1|9 7 1 1
F 5                1|7 7 5 0 = ·0000021715 = 5 F
                    1|9 6 1
G 5                 1|7 7 5 = ·0000002172 = 5 G
                      1|8 6
H 5                   1|7 8 = ·0000000217 = 5 H
                        |8
I 2                     |7 = ·0000000009 = I 2
                        1|
J 3                     1| = ·0000000001 = J 3
                    Take   ·0342672944
                    From  3·5502601816
Log. 3280·8992 = 3·5159928972
∴ log. 3·2808992 = 0·5159928972.
```

28. The constant sidereal year consists of 365·25636516 days; what is the log. of this number?

In this case it is better to bring *the constant* 35502601816 to 36525636516, instead of bringing the given number to the constant, as in the former examples.

```
                 3 5|5 0|2 6|0 1|8 1|6
                    |7 1|0 0|5 2|0 3|6
                          3 5|5 0|2 6|0
                 ------------------------
B 2 =            3 6 2|1 6 2|0 4 1|1 2 = ·0086427476 = 2 B
                    2|8 9 7|2 9 6|3 3
                          1 0|1 4 0|5 4
                                2 0|2 8
                 ------------------------
C 8 =            3 6 5 0|6 9 4 9|8 2 7 = ·0034726298 = 8 C
                      1|8 2 5 3|4 7 5
                                3|6 5 1
                 ------------------------
Take D 5 =       3 6 5 2 5 2 0 6 9 5 3 = ·0002171364 = 5 D
From             3 6 5 2 5 6 3 6 5 1 6
                 ------------------------
36525·2 )              |4 2 9 5 6 3
E 1 =                  |3 6 5 2 5 2 = ·0000043429 = 1 E
                       ------------
                        |6 4 3 1 1
F 1 =                   |3 6 5 2 5 = ·0000004343 = 1 F
                        ------------
                         2|7 7 8 6
G 7 =                    2|5 5 6 8 = ·0000003040 = 7 G
                         ------------
                           2|2 1 8
H 6 =                      2|1 9 1 = ·0000000261 = 6 H
                           ------------
I 0                          2|7
J 7 =                        2|5 = ·0000000003 = 7 J
                                   ------------
                                   ·0123376214
                              Add 3·5502601816
                                   ------------
Hence, log. 3652·5636516 = 3·5625978030
∴ log. 365·25636516 = 2·562597803.
```

29. M. Regnault determined with the greatest care, the density of mercury to be 13·59593 at the temperature 0°, centigrade. It is required to calculate the log. of 13·59593, to eight places of decimals.

In this case it is better to bring the given number to *the constant* 1371288574.

```
                 1 3 5|9 5 9|3 0 0
                    1|0 8 7|6 7 4
                          3|8 0 7
                                8
                 ------------------
C 8 =            1 3 7 0|5 0 7 8|9 = ·003472630 = 8 C
                        |6 8 5 2|5
                                1|4
                 ------------------
```

Subtract D 5 = 1 3 7 1 1 9 3 2 8 = ·000217136 = 5 D
From 1 3 7 1 2 8 8 5 7

9|5 2 9 = ·000026058 = E 6
E 6 = 8|2 2 7
1 3|0 2
F 9 = 1 2|3 4 = ·000003909 = F 9
6|8
H 5 = 6|9 = ·000000022 = H 5
Take ·003719755
From ·137128857
log. 1·359593 = ·133409102
∴ log. 13·59593 = 1·133409102.

30. M. Regnault finds the weight of a litre of air, under the parallel of latitude 45°, and at the same distance from the centre of the earth at which his experiments were made, to be 1·292697 grammes; what is the logarithm of this number?

The litre is 61·09908 *cubic inches, and the gramme is* 15·433159 *grains troy.*

1 2|9 2|6 9|7 0|0 0
|7 7|5 6|1 8|2 0
1|9 3|9 0|4 6
2|5 8|5 4
1|9 4
8

B 6 = 1 3 7 2 2 2 3 9 2 2 = ·0259282427 = 6 B.

31. We have here exceeded the constant 1371288574 to which we are working, but this is of little consequence, as we may now change our first intention and increase 1371288574 to 1372223922. This method of operating will often be found useful.

1 3 7 1|2 8 8 5|7 4
|8 2 2 7|7 3
2|0 6

D 6 = 1 3 7 2 1|1 1 5 5 3 = ·0002605637 = 6 D
1|0 9 7 6 9
1

E 8 = 1 3 7 2 2 2 1 3 2 3 = ·0000347434 = 8 E
F 1 = 1 3 7 2 = ·0000004343 = 1 F
G 8 = 1 0 9 8 = ·0000003474 = 8 G
H 9 = 1 2 3 = ·0000000391 = 9 H
I 4 = 6 = ·0000000017 = 4 I
·0002961296
Add ·1371288574
·1374249870

```
                          ·1374249870
Subtract   6 B =          ·0259282427
                          ----------
log. 1·292697 =           ·1114967443
```

32. The accurate and indefatigable M. Regnault in one of his experiments found that 1·36706 represents the volume of air at 100° centigrade thermom., the volume at 0° being supposed = 1. Compute the log. of 1·36706.

```
          1 3 6|7 0 6|0 0 0|0....A 0
               |1 1 0|1 1 8|0
                     |4 1 0|1....B 0
                           |1
          ---------------------
          1 3 7 1 1|6 5 2 8 2 = C 3
                 1|2 3 4 0 5
                            5     D 0
          ---------------------
          1 3 7 1 2 8 8 6 9 2 = E 9, which exceeds the
constant  1 3 7 1 2 8 8 5 7 4
          ---------------------
by                     1|1 8 which gives F 0, G 0, and
                       1|1 0 = H 8
                       -----
                          |8
                          |8 = I 6.
```

```
Then to the constant ·1371288574
                Add          347 = 8 H
                              26 = 6 I
                       ----------
                       ·1371288947
            Subtract       390863 = 9 E
                       ----------
                       ·1370898084
            Subtract     13022324 = 3 C
                       ----------
∴ log. 1·36706 =       ·1357875760
```

32. π = 3·14159265359, very nearly; what is its logarithm to thirteen decimal places?

This number may be brought to the constant 3550260181586591.

```
3 1 4 1 5 9 2 6 5 3 5 8 9 8
  3 1 4 1 5 9 2 6 5 3 5 9 0
-------------------------------
3 4|5 5|7 5|1 9|1 8|9 4|8 8 = A 1
   |6 9|1 1|5 0|3 8|3 7|9 0
   |   |3 4|5 5|7 5|1 9|1 9
-------------------------------
3 5 2|5 2 1|2 5 3|2 5 1|9 7 = B 2
   2|4 6 7|6 4 8|7 7 2|7 6
        7|4 0 2|9 4 6|3 2
           1 2|3 3 8|2 4
                1 2|3 4
-------------------------------
```

3 5 4 9 9 | 6 3 1 7 3 | 2 1 6 3 = C 7
2 | 8 3 9 9 7 | 0 5 3 9
9 | 9 3 9 9
2

3 5 5 0 2 4 | 7 1 8 0 2 1 | 0 3 = E 8
1 | 0 6 5 0 7 4 | 1 5
1 | 0 7

3 5 5 0 2 5 7 | 8 3 0 9 6 2 5 = F 3
2 | 1 3 0 1 5 4 7
5

Take 3 5 5 0 2 5 9 9 6 1 1 1 7 7 = G 6
From 3 5 5 0 2 6 0 1 8 1 5 8 6 6

2 | 2 0 4 6 8 9
2 | 1 3 0 1 5 6 = H 6

| 7 4 5 3 3
| 7 1 0 0 5 = I 2

3 | 5 2 8
3 | 1 9 5 = K 9

3 | 3 3
3 | 2 0 = L 9

1 | 3
1 | 1 = M 3

2 = N 6

·04139268515823 = 1 A (18.)
·00864274756529 = 2 B
·00303854235523 = 7 C
·00003474338483 = 8 E
·00000130288280 = 3 F
.....026057668 = 6 G
.....02605767 = 6 H
.....0086859 = 2 I
.....03909 = 9 K
.....0391 = 9 L
.....013 = 3 M
.....03 = 6 N

Take ·05311030889248
From 3·55026018158659

3·49714987269411 = log. 3141·5926535898,
And . . ·49714987269411 = log. 3·1415926535898.

33. This log. is correct to thirteen places. The logarithm of π

to fifty places is ·49714987269413385435126828829089887365167832438044.*

The labor of this computation can be somewhat abridged, (if the logs. of 113, 5, and 71 are known), by having recourse to the approximate ratio of Vieta, of the diameter of a circle to the circumference, *i. e.*, 113 : 355; which he derived from the pretended quadrature of Van Eick.

Thus $\pi \times 113 = 354{\cdot}999969855646466$, &c., which is readily brought to 355; hence the facility of the computation.

34. A mean synodic month is the interval between two successive conjunctions of the sun and moon, estimated according to their mean sidereal motions. The mean motion of the moon in $365\frac{1}{4}$ days, that is, in a Julian year, of 13 circumferences, 4 signs, 12° $39'$ $53{\cdot}3925''$. The mean motion of the sun in a Julian year is one circumference made less by $22{\cdot}584815''$. Hence, the relative motion of the sun and moon in the Julian year is 12 cir. 4 sig. 12° $40'$ $15{\cdot}977315''$ = 16029615·977315 seconds; what is the logarithm of this number?

In this case, raise 137128857423854 to 16029615977315.

| | | |

From 160296161518574
Take 160296159773150

.....1745424
1602962 = H 1

142462
128237 = J 8

14225
12824 = K 8

1401
1282 = L 8

119
112 = M 7

7
6 = N 4

·137128857423854 Const. (6.)
1 A = ·041392685158225
6 B = .25928242695858
1 C = ...434077479319
9 E = ...39086307936
4 G = ...173717788

·204923122782980 = the log. of 1·60296161518574.

Again:

1 H = ·000000004342945
8 J = ...347440
8 K = ...34744
8 L = ...3474
7 M = ...304
4 N = ...17

Take ·000000004728924
From ·204923122782980

·204923118054056 = log. 1·6029615977315.

And ∴ log. 16029615·977315 = 7·204923118054056 (4).

35. What is the logarithm of $365\frac{1}{4}$ to 14 decimal places? This, as before remarked, is the number of days in the Julian year: it is sometimes called the fictious year.

Bring 3|6|5|2|5|0|0|0|0|0|0|0|0|0 to 466924683287776 (4)
7|3|0|5|0|0|0|0|0|0|0|0|0|0
3|6|5|2|5|0|0|0|0|0|0|0|0

4 4|1 9|5 2|5 0|0 0|0 0|0 0|0 = A2
2|2 0|9 7|6 2|5 0|0 0|0 0|0
4|4 1|9 5|2 5|0 0|0 0|0
4|4 1|9 5|2 5|0 0|0
2|2 0|9 7|6 2|5
4 4|1 9|5

4 6 4|4 9 6|5 1 9|1 6 6|8 2 0 = B5
2|3 2 2|4 8 2|5 9 5|8 3 4
4|6 4 4|9 6 5|1 9 2
4|6 4 4|9 6 5
2|3 2 2
1

4 6 6 8|2 3 6 5|1 3 7 5|1 3 4 = C5
9 3 3 6|4 7 3 0|2 7 5
4 6 6 8|2 3 7

4 6 6 9 1|7 0 2 0 7|7 3 6 4 6 = D2
4 6 6 9 1|7 0 2 0 8

4 6 6 9 2 1|6 8 9 9 4 3|8 5 4 = E1
2|8 0 1 5 3 0|1 4 0
7|0 0 4

4 6 6 9 2 4 4|9 1 4 8 0 9 9|8 = F6
1|8 6 7 6 9 7 9|7
2|8

Take 4 6 6 9 2 4 6 7 8 2 5 0 8 2 3 = G4
From 4 6 6 9 2 4 6 8 3 2 8 7 7 7 6

. . . . . 5 0 3 6 9 5 3
4 6 6 9 2 4 7 = H1

3 6 7 7 0 6
3 2 6 8 4 7

4 0 8 5 9 = J7
3 7 3 5 4 = K8

3 5 0 5
3 2 6 8 = L7

2 3 7
2 3 3 = M5

4
4 = O9.

2 A = ·082785370316450
5 B = ·021606868913215
5 C = ..2170387396595
2 D = ...86854553726
1 E = ...4342923104

6 F = ...2605765590
4 G = ...173717788
1 H = ...4342945
7 J = ...304010
8 K = ...34744
7 L = ...3040
5 M = ...217
9 O = ...4

·106656608271428
4·669246832877758

4·562590224606330

∴ Log. $365\frac{1}{4}$ = 2·56259022460633 true to the last figure.

36. The sidereal period of Jupiter is 4332·58926673 days; what is the logarithm of this number to 14 places of decimals?

Bring 4 3|3 2|5 8|9 2|6 6|7 3|0 0|0 to 4669246 &c.
3|0 3|2 8|1 2|4 8|6 7|1 1|0
9|0 9|8 4|3 7|4 6|0 1|3
1 5|1 6|4 0|6 2|4 3|3
1 5|1 6|4 0|6 2|4
9|0 9|8 4|4
3|0 3|3
4

4 6 4|5 1 2|2 1 1|9 0 2|0 6 1 = B 7
3|3 2 2|5 6 1|0 5 9|5 1 0
4|6 4 5|1 2 2|1 1 9
4|6 4 5|1 2 2
2|3 2 3

4 6 6 8|3 9 4 2|2 7 3 1|1 3 5 = C 5
4 6 6 8|3 9 4 2|2 7 3

4 6 6 8 8|6 1 0 6 6|7 3 4 0 8 = D 1
3|7 3 5 0 8|8 8 5 3 4
1 3|0 7 2 8 1
2 6

4 6 6 9 2 3|4 5 8 8 6 9|2 4 9 = E 8
9 3 3 8 4 6|9 1 7
4 6 7

· · · · ·|4 3|9 2 7 1 6 6 3|3 = F 2
2|8 0 1 5 4 6 3|6
7|0

Take · · · · · ·|6 7 2 8 7 1 3 3 9
From 4 6 6 9 2 4 6 8 3 2 8 7 7 7 6 (6)

A B C D E F G 1|0 4 1 6 4 3 7 = G 6
9 3 3 8 4 9 4 = H 2

$$
\begin{array}{r|l}
1 & 0\,7\,7\,9\,4\,3 \\
 & 9\,3\,3\,8\,4\,9 = \mathrm{I}\,2 \\
\hline
1 & 4\,4\,0\,9\,4 \\
1 & 4\,0\,0\,7\,7 = \mathrm{J}\,3 \\
\hline
 & 4\,0\,1\,7 \\
 & 3\,7\,3\,5 = \mathrm{L}\,8 \\
\hline
 & 2\,8\,2 \\
 & 2\,8\,0 = \mathrm{M}\,6 \\
\hline
 & 2 \\
 & 2 = \mathrm{O}\,4 \\
\hline
\end{array}
$$

$$
\begin{array}{rl}
7\,\mathrm{B} = & \cdot 0302496164785 01 \\
5\,\mathrm{C} = & \ldots 2170387396595 \\
1\,\mathrm{D} = & \ldots\ldots 43427276863 \\
8\,\mathrm{E} = & \ldots\ldots 34743384832 \\
2\,\mathrm{F} = & \ldots\ldots\ldots 868588530 \\
6\,\mathrm{G} = & \ldots\ldots\ldots 260576682 \\
2\,\mathrm{H} = & \ldots\ldots\ldots\ldots 8685890 \\
2\,\mathrm{I} = & \ldots\ldots\ldots\ldots 868490 \\
3\,\mathrm{J} = & \ldots\ldots\ldots\ldots 130290 \\
8\,\mathrm{L} = & \ldots\ldots\ldots\ldots\ldots 3474 \\
6\,\mathrm{M} = & \ldots\ldots\ldots\ldots\ldots 261 \\
4\,\mathrm{O} = & \ldots\ldots\ldots\ldots\ldots\ldots 2 \\
\text{Take} & \cdot 032499313390510 \\
\text{From} & 4\cdot 669246832877758 \\
\hline
 & 4\cdot 636747519487248 = \log.\ 43325\cdot 8926673,
\end{array}
$$

and $\therefore$ log. $4332\cdot 58926673 = 3\cdot 636747519487248$.

37. It may be remarked here, that, if two permanent objects be placed on the surface of the earth, always retaining the same position, they may be so poised as to be situated in the same plane with the observer and a fixed star; although this coincidence may be but for an instant, yet this coincidence continually occurs, and the time elapsed between two consecutive coincidences is called the *sidereal day*. The *mean solar day* is the standard of time, and $366\cdot 25636516$ *sidereal days* $= 355\cdot 25636516$ *mean solar days*.

The *true solar day* is sometimes greater and sometimes less than the *mean solar day*.

While,

$$
\begin{array}{l}
\text{(sidereal day) : (mean solar day)} :: 365\cdot 25636516 : 366\cdot 25636516 \\
\qquad\qquad :: \cdot 997269672 : 1
\end{array}
$$

And,

$$
\text{(mean solar day) : (sidereal day)} :: 1 : 1\cdot 002737803;
$$

consequently 23 hours, 56 minutes, $4\cdot 0996608$ seconds of mean solar time $= 1$ sidereal day.

$\therefore$ 24 hours, 3 minutes, $56\cdot 5461797$ seconds, sidereal time $=$ a mean solar day.

The true solar day is the interval between two successive coin-

cidences of the sun with two fixed objects on the surface of the earth. The sun, the fixed objects, and the observer being always in the same vertical plane. *The sidereal year* is the time between two successive conjunctions of the sun with a fixed star.

This interval, at the commencement of the 19th century, was 365·256374417 *mean solar days*, equal 366·256374417 *sidereal days*.

There is another year, termed the *Anomalistic year*, of which we shall speak presently, when we determine the logarithm of 366·25636516 to fourteen places of decimals.

38. Our first object will be to bring 3550·26018158659 to 3662·5636516.

| | | |

3·550260181586591	=	constant.
·012964121347929	=	3 B
...434077479319	=	1 C
....86854553726	=	2 D
....39086307936	=	9 E
.....868588530	=	2 F
3·563786189864031	=	the log. of 3662·56372618119.

Then, 2 H	=	·000000008685890
3 J	=	130290
6 K	=	26058
3 L	=	1303
1 M	=	43
1 N	=	4
		.....8843588 take
		3·563785189864031 from
		3·563785181020443

Since there are but three places of whole numbers in the given number, we have log. 366·25636516 = 2·563785181020443.

39. The sun's relative orbit is a revolving ellipse, the motion of the transverse axis in t Julian years, reckoning from the commencement of the 19th century, is, according to Bessel, $11{\cdot}2936'' t + 0{\cdot}000081616'' t^2$; and in the space of one year between the t th and the $(t+1)$th the motion is $11{\cdot}293681616'' + 0{\cdot}000163232'' t$, which takes place in the same direction as the apparent motion of the sun. The motion of the sun in a sidereal year is one circumference; consequently, by the "*rule of three*" we have
$(360^\circ\ 0'\ 0'')$: $(360^\circ\ 0'\ 11{\cdot}293681616'' + 0{\cdot}000163232\,t)$ :: $(365{\cdot}25636516)$: $(365{\cdot}259548 + 0{\cdot}0000000459\,t)$ days; = the time in passing from the extremity of the transverse of its orbit when nearest the earth, to the same position in regard to the orbit of the earth, and is termed the *Anomalistic year*.

The length of the *Anomalistic year* for 1850 is 365·2592295 days:—the logarithm of 365·25636516 is known. (28.)

From	365·25922950
Take	365·25636516
	·00286434

This difference is too great to be diminished by repeated divisions; we shall therefore reduce the second number to the first in the usual manner.

```
ABCDEFGHIJKL
365256|365160
      2|556795
              8
-----------------
3652589|21963 = F7
       2|92207
-----------------
.....921|4170 = G8
        1|4610
-----------------
.......28|780 = H4
         |365
-----------------
.........91|45 = I1
          3|29
-----------------
.........47|4 J9
          2|6
-----------------
365259229500 = K7.
```

Log. 365·25636516	=	2·56259780300	(28.)
7 F	=	304006	
8 G	=	34744	
4 H	=	1737	
1 I	=	43	
9 J	=	39	
7 K	=	3	
		2·56260120872	the log. required.

I need scarcely observe, that with very little additional trouble the above logarithm may be computed to double the extent.

40. I shall now introduce several contrivances and ingenious contractions to abridge the foregoing direct and independent process; a process which in the most difficult case requires but little labor when compared to any former method of making logarithms.

A French metre is = 39·37079 English inches; required the log. of this number, the log. of 2 being known.

```
                 39·37079
The log. of             2 = .........·3010300
                 ----------
                 787|415|80
                    2|362|25
                         2|36
                 ----------
                 78978041 = 3 C =  ..13022 add
Constant         78974890          ·3023322
                 ----------
                 ....3|151
                     3|159 = 4 E =  ....174 subtract
                 ----------          ·3023148 take
                                     ·8974890 from
                                     ---------
                 ∴ log. 39·37079 = 1·5951742
```

For none of the usual purposes of calculation is it necessary to compute logarithms to a greater extent than above, but the logarithms of the fundamental numbers on which the actual construction of a table of logarithms depend, must be carried to a greater extent of decimals than those tabulated, in order to provide against the errors that would affect the latter figures of many logarithms found by the combination of others.

41. To find the logarithm of 19.

Log. 237·58120876 = 2·3758120876
log. 8 = ·90308998 70
log. 1900·64967... = 3·2789020746

Bring 1 9 0 0|0 0 0 0|0 to 190054967
5 7 0 0|0
6

Take 1 9 0 0 5 7 0 0 6 = 3 D = ·000130282
From 1 9 0 0 6 4 9 6 7

. . . . .|7 9 6 1
7 6 0 2 = 4 E = ...17372

·|3 5 9
1 9 0 = 1 F = ...434

1|6 9
1|5 2 = 8 G = ...347

1|7
1|7 = 9 H = ...39

·000148474 subtract
3·278902075

∴ log. 1900· = 3·278753601
Hence log. 19· = 1·278753601

This log. is true even in the last figure; for the log. of 19 to 20 places is 1·27875300952828961536.

42. Let it be required to find the log. of 3, 9, 27, 81, &c. $3^{\frac{m}{n}}$

First, $3^2 = 9$.....

9.....

99——————— = 1 A = ·041392685158225
99

9999....——————— = 1 B = 004321373782643
9999

99999999..... = 1 D = 000043427276863
99999999 = 1 H = ...4342945

·045757490560676

From the Constant ·999999999999999 (6)
Take ·045757490560676

2) ·954242509439323 = log. 9
·477121254719662 = log. 3 or $\sqrt{9}$
Log. 3 × 3 = 1·431363764158986 = log. 27 or 3^3
Log. 3 × 4 = 1·908485018878648 = log. 81 or 3^4
Log. 3 × 5 = 2·385606273598312 = log. 243 or 3^5
&c. = &c. = &c.

The logarithm of 3, or any power of 3, can in this manner be accurately determined to a considerable number of decimal places; to 21 places it is ·477121254719662437295.

43. Required the logarithm of 99, 11, 121, &c.

99..
99.... = B 1
9999..... = D 1
99999999 = H 1
Then B + D + H = ·004364805402451.

From the constant ·999999999999999 (6)
Take ·004364805402451

·995635194597548 = log. 9·9
log. 9 = ·954242509439323 by (42)

1·041392685158225 = log. 11 or $\frac{99}{9}$.

Hence the log. of 121, 1331, 14641, &c., to any power of 11 can be speedily obtained. From the two last examples the log. of any composite number made up by the factors, 3 and 11, is readily deduced, as 3 × 11 = 33; 27 × 11 = 297; 81 × 11 = 891, &c.

44. Required the log. of 999, 111, 37, &c.

By bringing 999 to 999999999, &c., gives 1 C, 1 E, and 1 L.
Then C = ·000434077479319
F = 434294265
L = 434

Take ·000434511774018
From Constant ·999999999999999

2·999565488225981 = log. 999
log. 9 = ·954242509439325 (42)

2·045322978786656 = log. 111
log. 3 = ·477121254719662 (42)

1·568201724066994 = log. 37.

The log. of 37 to 20 places is 1·56820172406699499681.

From the foregoing calculations it will be readily perceived, with what speed and accuracy the log. of any number expressed by a repetition of 9 or 1 can be obtained.

In a similar manner the lag. of 101, 1001, 10001, 100001, &c., may be found: for $\frac{9999}{99} = 101$; $\frac{999999}{999} = 1001$; $\frac{99999999}{9999} = 10001$, &c.

The log. of 5 may be found by taking the log. of 2 from the log. of 10.

The log. of 7 may be determined from 49, which will readily converge to 50.

The log. of 13 is computed with great ease, since $\frac{1001}{11 \times 7} = 13$.

45. What is the log. of 342·657.

3 4 2 | 6 5 7 | . . .	
3 4 2 | 6 5 7	
3 4 2 9 9 9 | 6 5 7	C 1
| 3 4 2 9 9 9 6 5 7	F 1
3 4 2 &c.	L 1
3 4 2·9 9 9 9 9 9 9 9 9 9 9 9 &c. very nearly = 343	

3 log. 7	= 2·535294120042770	= log. 343
1 C + 1 F + 1 L	= ·000434511774018	(44.)
Log. 342·657	= 2·534859608268752	

The number 342657 is of a peculiar formation, the last three digits being the complements of each of the first three of 9. The logarithms of such numbers may be calculated with great rapidity, when the log. of the first half of the digits *plus one*, is known.

46. The number 1·2345679 has also a peculiarity which renders the computation of its logarithm remarkably simple.

Log. 9·99999999	= 0·999999999565705	= log. 10 — I
log. 8·1	= ·908485018878650	= log. 3^4
log. 1·2345679	= ·091514980687055	

Then, log. 12345679 + log. 9 = log. 111111111
log. 12345679 + log. 18 = log. 222222222
log. 12345679 + log. 27 = log. 333333333
log. 12345679 + log. 36 = log. 444444444
log. 12345679 + log. 45 = log. 555555555
log. 12345679 + log. 54 = log. 666666666
log. 12345679 + log. 63 = log. 777777777
log. 12345679 + log. 72 = log. 888888888
log. 12345679 + log. 81 = log. 999999999

TO DETERMINE THE NUMBER CORRESPONDING TO A GIVEN LOGARITHM.

This problem has been very much neglected, so much so, that none of our elementary books ever allude to a method of computing the number answering to a given logarithm. When an operation is performed by the use of logarithms, it is very seldom that the resulting logarithm can be found in the table; we have therefore to find the nearest less logarithm, and the next greater, and correct them by proportion, so that there may be found an intermediate number that will agree with the given logarithm, or nearly so. But although the *proportional parts of the difference* abridge this process, we can only find a number appertaining to any logarithm to seven places of figures when using our best modern tables. As, however, the tabular logarithms extend only to a degree of approximation, fixed generally at seven decimal places, all of which, except those answering to the numbers 10 and its powers, err, either in excess or defect, the maximum limit of which is $\frac{1}{2}$ in the last decimal, and since both errors may conspire, the 7th figure cannot be depended on as strictly true, unless the proposed logarithm falls between the limits of log. 10000 and log. 22200.

Indubitably, we are now speaking of extreme cases, but since it is not an unfrequent occurrence that some calculations require the most rigid accuracy, and many resulting logarithms may be extended beyond the limits of the table, this subject ought to have a place in a work like the present. It is not part of the present design to enter into a strict or formal demonstration of the following mode of finding the number corresponding to a given logarithm, as the operation will be fully explained by suitable examples.

47. What number corresponds to the logarithm 3·44496555.

The next less constant log. to the one proposed is 2·37581209, or rather, 3·37581209, when the characteristic or index is increased by a unit.

First from 3·44496555

take 3·37581209 (6.) .....

·06915346

·04139269 = 1 A

·02776077

·02592824 = 6 B

..183253

173631 = 4 C

Secondly.

2 | 3 7 5 8 1 2 0 9 constant

2 | 3 7 5 8 1 2 1 = A 1

2 6 | 1 3 | 3 9 | 3 3 | 0

1 | 5 6 | 8 0 | 3 6 | 0

3 | 9 2 | 0 0 | 9

5 | 2 2 | 7

3 | 9

....9622		2 7 7 | 4 1 6 | 9 6 5	= B 6
8685	= 2 D	1 | 1 0 9 | 6 6 8	
.....937		1 | 6 6 4	
869	= 2 E	1	
......68		2 7 8 5 | 2 8 2 9 | 8	= C 4
43	= 1 F	5 5 7 0 | 6	
.....25		3	
22	= 5 G	2 7 8 5 8 | 4 | 0 | 0 | 7	= D 2
.....3		5 | 5 | 7 | 2	= E 2
3	= 7 H	2 | 7 | 9	= F 1
		1 | 3 | 9	= G 5
		1 | 9	= H 7
		2 7 8 5 9 0 0 1 6	

∴ 2785·90016 is the number sought.

48. What number corresponds to the logarithm 5·73437574?

When the index of this log. is reduced by a unit, the nearest next less constant is 4·66924683.

From 4·73437574		
Take 4·66924683		
.6512891		
4139269	.....1 A	
.2373622		
2160687	.....5 B	
..212035		
173631	.....4 C	
...39304		
39085	.....9 D	
.....219	There is	neither the equal of
217	.....5 F	this number, nor a
.....2	.....0 G	less, obtainable from
2	.....4 H	E, ∴ E 0, or E, is
		omitted.

Then, 4 | 6 6 9 2 4 6 8 3
4 6 6 9 2 4 6 8 .....A 1

5 1	3 6	1 7	1 5	1			
2	5 6	8 0	8 5	8			
	5	1 3	6 1	7			
		5	1 3	6			
			2	6			

```
5 3 9|8 1 6|7 8 8.....B 5
   2|1 5 9|2 6 7
      3|2 3 9
         2
-------------------
5 4 1 9|7 9 2 9|6.....C 4
     4|8 7 7 8|1
        1 9|5
-------------------
5 4 2 4 6|7|2|7|2.....D 9
         |2|7|1|2.....F 5
           |2|2.....H 4
-------------------
5 4 2 4 7 0 0 0 6
```

∴ 542470·006 is the number whose logarithm is 5·73437574.

49. Had the given logarithm represented a decimal with a positive index, the required number would be 0·000054247, &c.; or if written with a negative index, as $\bar{5}$·73437574, the result would be the same, for the characteristic $\bar{5}$, shows how many places the first significant figure is below unity.

50. Required the number corresponding to log. 2·3727451.

The constant 100000000 is the one to be employed in this case.

```
1·3727451  the given log. minus 1 in the index.
1·0000000
---------
·3727451
 3725342.....9 A
 -------
...2109
   1737.....4 D
   ----
....372
    347.....8 E
    ---
.....25
     22.....5 F
     --
      3
      3.....7 G
      -

1|0|0|0|0|0|0|0 Constant
 |9|0|0|0|0|0|0
 3|6|0|0|0|0|0
   8|4|0|0|0|0
   1|2|6|0|0|0
     1|2|6|0|0
        8|4|0
          3|6
           |9
-----------------
```

2 3 5 7	9 4 8 5	A 9
	9 4 3 2	
	1	
2 3 5 8 8	9 1 8	D 4
	1 8 9 7	E 8
	1 1 8	F 5
	1 6	G 7
2 3 5 9 0 9 4 9		

∴ 235·90949 is the required number, and the seconds in the diurnal apparent motion of the stars.

$$235{\cdot}90949'' = 3' \; 55{\cdot}90949''.$$

51. Let it be required to find the *hyperbolic* logarithm of any number, as 3·1415926536. The common log. of this number is ·49714987269 (33), and the common log. of this log. is $\bar{1}$·6964873.

The modulus of the common system of logarithms is ·4342944819, &c.

∴ 1 : ·4342944819 :: hyperbolic log. N : common log. N.

To distinguish the hyperbolic logarithm of the number N from its common logarithm, it is necessary to write the hyp. log. Log. N, and the common logarithm log. N.

Hence, 4342944819 × Log. N = log. N;

or log. (·4342944819) + log. (log. N) = log. (log. N).

∴ log. (Log. N) = log. (log. N) — $\bar{1}$·6377843; for $\bar{1}$·6377843 = log. ·4342944819.

Now, to work the above example, from $\bar{1}$·6964873

take $\bar{1}$·6377843

·0587030, the number corresponding to this *com. log.* will be the *hyp. log.* of 3·1415927. ·0587030 must be reduced to ·0000000 which is known to be the log. of 1.

·0587030	
0413927	1 A
.173103	
172855	4 B
....248	
217	5 E
.....31	
30	7 F
.....1	2 G

1 A =	1 1	0 0	0 0	0 0	0				
	4 4	0 0	0 0	0 0	0				
		6 6	0 0	0 0	0				
				4 4	0				
					1				
	1 1 4 4 6		6	4	4	1	=	B 4	
			5	7	2	3	=	E 5	
			8	0	1	=	F 7		
				2	3	=	G 2		
	1 1 4 4 7 2 9 8 8								

∴ 1·14472988 is the hyperbolic log. of 3·1415927, true to the last figure; for the hyp. log. 3·1415926535898 = 1·1447298858494.

The reason of this operation is very clear, because

$1 \times 1{\cdot}1 \times (1{\cdot}01)^4 \times (1{\cdot}00001)^5 \times (1{\cdot}000001)^7 \times (1{\cdot}0000001)^2 = 1{\cdot}14472988.$

52. This example answers the purpose of illustration, but the hyp. log. of 3·1415927 can be more readily found by dividing its com. log. ·49714987269 by the constant ·4342944819, which is termed the Modulus of the common system of logarithms.

53. Suppose it is known that $\bar{1}{\cdot}3426139$ is the log. of the decimal which a *French litre* is of an English gallon. Required the decimal.

The index, $\bar{1}$, may be changed to any other characteristic, so as to suit any of *the constants* (6), as the alteration is easily allowed for when the work is completed. In this instance it is best to put $+ 1$ instead of $\bar{1}$.

From 1·3426139		1|0|0|0|0|0|0|0|0 Constant
Take 1·0000000		8|0|0|0|0|0|0|0
·3426139		2|8|0|0|0|0|0|0
3311415	= 8 A	5|6|0|0|0|0|0
·0114724		7|0|0|0|0|0
..86427	= 2 B	5|6|0|0|0
28297		2|8|0|0
26045	= 6 C	8|0
2252		1
2171	= 5 D	2 1|4 3|5 8|8 8|1 = A 8
81		4 2|8 7|1 7|8
43	= 1 E	2 1|4 3|6
38		2 1 8|6 6 7|4 9 5 = B 2
35	= 8 F	1|3 1 2|0 0 5
3		3|2 8 0
3	= 7 G	4
		2 1 9 9|8 2 7 8|4 = C 6
		1|0 9 9 9|1
		2|2
		2 2 0 0 9|2|7|9 7 = D 5
		2|2|0 1 = E 1
		1|7|6 1 = F 8
		1|5 4 = G 7
		2 2 0 0 9 6 9 1 3

∴ The French litre = ·2200969 English gallons.

54. In measuring heights by the barometer, it is necessary to know the ratio of the density of the mercury to that of the air.

At Paris, a *litre* of air at 0° centigrade, under a pressure of 760 millimetres, weighs 1·293187 grammes. At the level of the sea, in latitude 45°, it weighs 1·292697 grammes. A *litre* of water, at its maximum density, weighs 1000 grammes, and a *litre* of mercury, at the temperature of 0° cent., weighs 13595·93 grammes:

$$\therefore \frac{13595{\cdot}93}{1{\cdot}292697} = \text{the ratio at } 45^\circ$$

Now, log. 13595·93 = 4·133409102 (29)
and log. 1·292697 = 0·111496744 (30)

4·021912358 = the log. of the ratio at 45°.

To find the number corresponding to this log., it is necessary to reject the index for the present, and reduce the decimal part to zero. By this means the necessity of using any of the constants (6) is superseded.

·021912358	
·021606869	= 5 B
...305489	
303991	= 7 D
.....1498	
1303	= 3 F
.....195	
174	= 4 G
.....21	
17	= 4 H
4	
4	= 9 I

1 0 | 0 0 | 0 0 | 0 0 | 0 0	
5 0 | 0 0 | 0 0 | 0 0	
1 | 0 0 | 0 0 | 0 0	
1 | 0 0 | 0 0	
5 0	
1 0 5 1 | 0 1 0 0 | 5	= B 5
7 3 5 7 | 1	
2 | 2	
1 0 5 1 7 4 | 5 | 9 | 8	= D 7
3 | 1 | 6	= F 3
4 | 2	= G 4
4	= H 4
1	= I 9
1 0 5 1 7 4 9 6 1	

$\therefore$ by logarithms, $\frac{13595{\cdot}93}{1{\cdot}292697}$ = 10517·49, &c., which is easily verified by common division.

55. M. Regnault found that, at Paris, the *litre* of atmospheric air weighs 1·293187 grammes; the *litre* of nitrogen 1·256167 grammes; a *litre* of oxygen, 1·429802 grammes; of hydrogen, 0·089578 grammes; and of carbonic acid, 1·977414 grammes. But, strictly considered, these numbers are only correct for the locality in which the experiments were made; that is for the latitude of 48° 50′ 14″ and a height about 60 metres above the level of the sea; M. Regnault finds the weight of the *litre* of air under the parallel of 45° latitude, and at the same distance from the centre of the earth as that which the experiments were tried to be 12·926697.

Assuming this as the standard, he deduces for any other latitude, any other distance from the centre of the earth the formula.

$$w = \frac{1{\cdot}292697\ (1{\cdot}00001885)\ (1-0{\cdot}002837)\cos.\ 2\lambda}{1+\frac{2h}{\mathrm{R}}}$$

Here, w is the weight of the litre of air, R the mean radius of the earth = 6366198 metres, h the height of the place of observation above the mean radius, and λ the latitude of the place.

56. At Philadelphia, lat. 39° 56′ 51·5″, suppose the radius of the earth to be 6367653 metres, the weight of the *litre* of air will be 1·2914392 grammes. The ratio of the density of mercury to that of air at the level of the sea at Philadelphia is 10527·735 to 1; required the number of degrees in an arc whose length is equal to that of the radius.

As 3·1415926535898 : 1 :: $\frac{360}{2}$: the required degrees.

Log. 360 = 2·556302500767
log. 3·14159265359 = 0·497149872694
2·059452623073
log. 2 = 0·301029995664
1·758122632409 = the log. of the number required.

When the index of this log. is changed into 4, the nearest next less constant is 4·669246832878.

From 4·758122632409
Take 4·669246832878
·088875799531

2 A = .82785370316
..6090429215
1 B = 4321373783
..1769055432
4 C = 1736309917
....32745515
7 E = 30400462
.....2345053
5 F = 2171471
.....173582
3 G = 130288
.....43294
9 H = 39087
.....4207
9 I = 3909
.....298
6 J = 261
.....37
8 K = 35
.....2
5 L = 2

4|6|6|9|2|4|6|8|3|2|8|7|8 = Const.
9|3|3|8|4|9|3|6|6|5|7|6
4|6|6|9|2|4|6|8|3|2|9·

5 6|4 9|7 8|8 6|6 7|7 8|3 = A 2
5 6|4 9|7 8|8 6|6 7|8

5 7 0|6 2 8|6 5 5|4 4 6|1 = B 1
2|2 8 2|5 1 4|6 2 1|8
3|4 2 3|7 7 1|9
2|2 8 2|5
6

5 7 2 9 1|4 5 9 6 1|2 2 9 = C 4
4|0 1 0 4 0|2 1 7
1 2|0 3 1

5 7 2 9 5 4|7 0 1 3 4 7|7 = E 7
2|8 6 4 7 7 3|5
5|7

5 7 2 9 5 7 5|6|6|1|2|6 9 = F 5
1|7|1|8|8|7 3 = G 3
5|1|5|6|6 2 = H 9
5|1|5|6|6 = I 9
3|4|3 8 = J 6
4|5 8 = K 8
2 9 = L 5

5 7 2 9 5 7 7 9 5 1 2 9 5 = the number required.

But the original index is 1; ∴ 57·29577951295° are the num-

ber of degrees in an arc the length of which is equal to that of the radius.

57. The above result may be easily verified by common division, a method, no doubt, which would be preferred by many, for logarithms are seldom used when the ordinary rules of arithmetic can be applied with any reasonable facility. However, this example, like many others, is introduced to show with what ease and correctness the number corresponding to a given log. can be obtained. The extent, also, by far exceeds that obtainable by any tables extant.

Other computations give,

$$r^\circ = 57{\cdot}2957795131^\circ = 57^\circ\ 17'\ 44''\ {\cdot}80624$$

the degrees in an arc = radius.

$$r' = 3437{\cdot}7467707849' = 3437'\ 44''\ {\cdot}80624$$

the minutes in an arc = radius.

$$r'' = 206264{\cdot}8062470963''$$

the number of seconds in an arc = radius.

58. The relative mean motion of the moon from the sun in a Julian or fictious year, of $365\frac{1}{4}$ days, is 12 cir. 4 signs, $12^\circ\ 40'\ 15{\cdot}977315' = 16029615{\cdot}977315''$.

$\therefore$ 16029615·977315″ : 1 circumference (= 129600″)
:: 365·25 days
: 29·5305889216 days = the mean synodic month.

This proportion may, for the sake of example, be found by logarithms.

Log. 365·25.....	2·56259022460634
log. 1296000.....	6·11260500153457
	8·67519522614091
log. 16029615·977315 =	7·20492311805406
	1·47027210808685

If the index of this log. be made 2 instead of 1, the nearest next less constant will be 2·375812087593221.

From	2·47027210808685			
Take	2·37581208759322	..	2 3 7 5 8 1 2 0 8 7 5 9 3 2 2	Const.
	·09446002049363		4 7 5 1 6 2 4 1 7 5 1 8 6 4	
2 A =	08278537031645		2 3 7 5 8 1 2 0 8 7 5 9 3	
	.1167465017718		2 8 7 4 7 3 2 6 2 5 9 8 7 7 9	= 2 A
2 B =	864274756529		5 7 4 9 4 6 5 2 5 1 9 7 6	
	..303190261189		2 8 7 4 7 3 2 6 2 6 0	
6 C =	260446487591		2 9 3 2 5 1 4 7 5 1 7 7 0 1 5	= 2 B
	...42743773598		1 7 5 9 5 0 8 8 5 1 0 6 2	
9 D =	39084549177		4 3 9 8 7 7 2 1 2 8	
	3659224421		5 8 6 5 0 2 9	
8 E =	3474338483		4 3 9 9	
			2	

	184885938	2 9 5 0 | 1 5 3 8 | 8 6 6 9 | 6 3 5 = C 6
4 F =	173717706	2 | 6 5 5 1 | 3 8 4 9 | 8 0 3
	11168232	1 0 | 6 2 0 5 | 5 4 0
2 G =	8685889	2 4 | 7 8 1
	2482343	4
5 H =	2171473	2 9 5 2 8 | 1 0 0 8 7 | 4 9 7 6 3 = D 9
	310870	2 | 3 6 2 2 4 | 8 0 7 0 0
7 I =	304006	8 | 2 6 7 8 7
	6863	1 7
1 J =	4343	2 9 5 3 0 4 | 6 3 2 0 5 7 | 2 6 7 = E 8
	2520	1 | 1 8 1 2 1 8 | 5 2 8
5 K =	2172	1 | 7 7 2
	348	2 9 5 3 0 5 8 | 1 3 2 7 7 5 6 | 7 = F 4
8 L =	347	5 9 0 6 1 1 6 | 3
2 N =	1	3
		2 9 5 3 0 5 8 7 | 2 | 3 | 3 | 8 | 7 3 | 3 = G 2
		1 | 4 | 7 | 6 | 5 | 2 9 | 4 = H 5
		2 | 0 | 6 | 7 | 1 4 | 1 = I 7
		2 | 9 | 5 3 | 1 = J 1
		1 | 4 | 7 6 | 5 = K 5
		2 | 3 6 | 2 = L 8
		6 = N 2
		2 9 5 3 0 5 8 8 9 2 1 7 8 3 2

∴ 29·5305889218 is the number required.

59. To perform, by logarithms, the ordinary operations of multiplication, division, proportion, or even the extraction of the square root, except in the way of illustration, is not the design of these pages; for such an application of logarithms, in a particular manner only, diminish the labor of the operator. It is not necessary, however, to examine minutely here the instances in which common arithmetic is preferable to artificial numbers; besides, much will depend on the skill and facility of the operator. But when it is required to find the numerical value of N, in such an expression as

$$N = \sqrt[\varepsilon^{9}]{Y^{\pi^{7}}}$$

the use of logarithms becomes indispensable to obtain a result in any reasonable time.

60. To exemplify this, let Y be the number of days in the *mean solar*, or *mean equinoctial year* at the present time, namely, 1849; for this year is now diminishing.

$\pi = 3{\cdot}141592653589793$

$\varepsilon = 2{\cdot}718281828459045$ &c., = the number whose hyp. log. is unity.

The common log. of $\varepsilon = {\cdot}43429448190325182765112891891 6$.

Before finding $Y^{\frac{\varepsilon^9}{\pi^7}} = N$, correct to 15 places of figures, it may not be unnecessary to describe, or rather define, the interval of time called the *mean solar* or *mean equinoctial year*. The equinoctial points have a retrograde motion, that is, a motion contrary to the apparent motion of the sun in its path; the amount of this motion is given by Laplace, in his *Mécanique Céleste*, vol. iii. pp. 112 and 158. Bessel reduces it to

$$50{\cdot}2235'' \, t + 0{\cdot}0001221805'' \, t^2 + 0{\cdot}000000000215'' \, t^3.$$

t being the number of fictious years, of $365\frac{1}{4}$ days, reckoned from the commencement of the 19th century.

To find the motion from t years to $t + 1$ years from the commencement of the 19th century. Bessel finds the mean equinoctial year in days to be $365{\cdot}24222013 \mp {\cdot}00000006686 \, t$; t denoting the fictious years, of $365\frac{1}{4}$ days, counted from the beginning of the year 1800.

This result was established by putting $t + 1$ for t, whenever t occurred, the result being made less by the above motion.

The sign indicating subtraction is to be used for any years after 1800, and the sign of addition is to be employed for time before 1800.

Thus, $1849 - 1800 = 49$
and $\cdot00000006686 \times 49 = \cdot00000327614$
then, from $365{\cdot}24222013\ldots$
take $\cdot00000327614$
$365{\cdot}24221685386$ = the number of days in the *mean solar year* at the present time.

Let n be any number of mean equinoctial years,
$a = 365{\cdot}24222013$, and
$b = \cdot00000006686$, the sum of the series,

$$(a - b) + (a - 2b) + (a - 3b) \ldots\ldots \left\{ a - (n - 1)\, b \right\} =$$

$$\frac{n}{2} \left\{ 2a - nb \right\} = an - \frac{b}{2} n^2.$$

Consequently, $365{\cdot}24222013 \, n \mp {\cdot}00000003343 \, t^2$ gives the number of days from the beginning of the year 1800; n, as before remarked, denoting the number of mean equinoctial years.

$(-)$ is to be taken for the time after 1800; and $(+)$ for the time before 1800. From the above formula our calendar is formed and corrected.

$$\text{Log. N} = \frac{\varepsilon^9}{\pi^7} \text{ log. Y.}$$

$$\text{Log. (log. N)} = 9 \text{ log. } \varepsilon + \text{log. (log. Y)} - 7 \text{ log. } \pi$$

Log. Y, or log. $365{\cdot}24221685386 = 2{\cdot}5625809700863101$ and log. (log. Y) or log. $2{\cdot}5625809700863101 = {\cdot}408677586719873$.

Then, to log. (log. Y) = 0·408677586719873 add
9 log. ε = 3·908650337129266
4·317327923849139
Subtract 7 log. π = 3·480049108858937 (33)
0·837278814990202 = (log. N)

By changing the index, 0, into 6, and knowing that log. 6834720·776754357 = 6·834720776754357 (6), the work will stand in the usual manner.

From 6·837278814990202
Take 6·834720776754357

..2550038235045
5 C = 2170387396595
...387650839250
8 D = 347418214864
....40232624446
9 E = 39086307936
.....1146316510
2 F = 868588530
.....277727980
6 G = 260576682
.....17151298
3 H = 13028835
.....4122463
9 I = 3908655
.....213808
4 J = 173720
.....40088
9 K = 39087
.....1001
2 L = 869
.....132
3 M = 130
.....2
5 O = 2

..6 8 3|4 7 2|0 7 7|6 7 5|4 3 5|7
3|4 1 7|3 6 0|3 8 8|3 7 7|2
6|8 3 4|7 2 0|7 7 6|8
6|8 3 4|7 2 0|8
3|4 1 7|4
7

6 8 6 8|9 6 2 7|9 6 2 2|7 2 8 6 = C 5
5|4 9 5 1|7 0 2 3|6 9 8 2
1 9|2 3 3 0|9 5 8 3
3 8|4 6 6 2
4 8

6 8 7 4 4|5 9 8 9 0|1 5 8 5 6|1 = D 8
6|1 8 7 0 1|3 9 0 1 1|4
2 4|7 4 8 0 5|6
5 7|7

6 8 7 5 0 7|8 6 1 6 2 9|7 3 0 8 = E 9
1|3 7 5 0 1 5|7 2 3 3
6 8 7 5

6 8 7 5 0 9 2|3 6 6 4 6 1 4|1 6 = F 2
4|1 2 5 0 5 5 4|2 0
1 0|3 1

6 8 7 5 0 9 6 4|9 1 5 1 7 8 6 7 = G 6
2|0 6 2 5 2 8 9 5
2

6 8 7 5 0 9 6 6 9|7|7|7|0|7 6|4 = H 3
6|1|8|7|5|8 7|0 = I 9
2|7|5|0|0 3|9 = J 4
6|1|8|7 5|9 = K 9
1|3|7 5|0 = L 2
2|0 6|3 = M 3
3|4 = O 5

Hence, log. N = 6·8 7 5 0 9 6 7 6 3 0 3 1 2 7 9

The index of this logarithm need not be changed, since it so nearly corresponds to the constant 6·834720776754357 (6).

From 6·875096763031279
Take 6·834720776754357

.40375986276922
9 B = 38892364043787
..1483622233135
3 C = 1302232437957
...181389795178
4 D = 173709107452
.....7680687726
1 E = 4342923104
.....3337764622
7 F = 3040059855
.....297704767
6 G = 260576682
.....37128085
8 H = 34743560
.....2384525
5 I = 2171475
.....213050
4 J = 173720
.....39330
9 K = 39087
.....243
5 M = 217
.....26
6 N = 26

| | | |

this extent, to find them to 48 places of figures would require but little additional labor.

To be independent of tables, we must remember 16 numbers, of which are the eight following:—

$$\cdot041392685158225 = A$$
$$\cdot004321373782643 = B$$
$$\cdot000434077479319 = C$$
$$\cdot000043427276863 = D$$
$$\cdot000004342923104 = E$$
$$\cdot000000434294265 = F$$
$$\cdot000000043429447 = G$$
$$\cdot000000004342945 = H$$

62. It is easily observed that A, the first letter has one cipher; B, the second letter, has two ciphers; C, the third letter, has three ciphers; and so on. Hence, if I wish to know the value of K, it being the eleventh letter, I set down eleven ciphers, and as many of the figures 4342945 as will complete the fifteen figures. When the proper allowance is made,

$$\cdot000000000004343 = K$$

Consequently, when all the numbers up to H are known, the succeeding ones are easily found. After setting down the proper number of ciphers, it is readily remembered that the next figure is 4 in all cases; then the only difficulty to be overcome is to remember the succeeding figures.

63. In this case, artificial memory completely answers the purpose, and the following scale must be committed to memory:—

1	2	3	4	5	6	7	8	9	0
l	*n*	*m*	*r*	*f*	*b*	*t*	*s*	*p*	*d*

l is put for 1, because one line forms *l*; *n* is put for 2, because two lines form *n*; *m* is put for 3, because three lines form *m*. *r* is made to represent 4, because it is a very prominent letter in the *word four*; *f* is put for 5, because it is a very prominent letter in the word *five*; *b* is put for 6, because they resemble each other in shape; *t* is put for 7, and *s* for 8, for the same reason; *p* is substituted for 9, and *d* for 0, without any reason for so doing, except that they are easily employed to form words. According to this gamut the word

l	o	g	a	*r*	i	*t*	h	*m*	*s*
1				4		7		3	8

represents the number 14738, for the letters o, g, a, i, h, are without numeral representatives.

In committing numbers to memory by this plan I am obliged to

employ a great deal of *jargon*, but not useless jargon. However, I shall be as sparing of the commodity as possible, as there is more demand for it in other sciences, and in *higher places*.

64. A = ·04 *limping*, *bashful*, *fashion*, *unfix*.

65. B = ·004 *manly*, *mathematics*, *new broom*.

66. C = ·0004 *more*, *ditto*, *art*, *poem*, *loop*.

67. D = ·00004 *margin tantivy bask beam*.

68. E = ·000004 *margin*, *pen-moulder*.

69. F = ·0000004 *margin*, *apron*, *beef*.

70. G = ·00000004 *margin*, *parrot*.

71. H = ·000000004 *margin*, *proof*.

The second *margin* or 342 being part of D, E, F, G, H, is easily remembered.

72. *Limit lines. softer names. foreign.*
1 3 7 1 2 8 8 5 7 4 2 3 8 5 4 2

73. *Inmate of islands, thief pam, rarefy.*
2 3 7 5 81 208 7 5 9 3 4 4 5

It may here be observed, that pam is a name given to the knave of clubs.

74. *Muff do nobody closely. fish, beef, peel.*
3 55 0 2 6 0 1 8 1 5 8 6 5 9 1

75. *Rabbi open your bosom. noise, tattoo, foes.*
4 66 9 2 4 6 8 3 2 8 7 7 7 5 8

76. *Fat boy, dwarf. big pomeroy, low muffin too.*
5 7 6 0 45 6 9 3 4 1 3 55 2 7

77. *Be smart and that, but from fat.*
6 83 47 20 7 7 6 7 54 3 5 7

78. *The sceptre soaped. my lamps larry.*
7 8 974 8 9 0 3 1 398 1 44

79. *Spy, fly, ply, fop; poison by this map.*
89 51 91 5 9 9 8 2 6 7 8 3 9

As it is often convenient, the modulus of the common system of logarithms may be added.

81. The angle contained between the plane of the equator and the elliptic is denominated the obliquity of the elliptic, and is shown from repeated observations to be variable, and continually decreasing.

At the present time, namely, 1849, this angle is
23° 27′ 33·87243″; required its log. sine.

60) 33·87243

·5645405 = the decimal parts of a minute.

∴ 23° 27′ 33·87243″ = 1407·5645405′.

An arc of 1′ to radius 1 = ·00029088820866572159615394846 14; which when multiplied by 1407·5645405′, gives ·409443927767435, the length of an arc of 23° 27·5645405′.

Put x = ·409443927767435; and because,

$$\text{Sine } x = x - \frac{x^3}{1.2.3} + \frac{x^5}{1.2.3.4.5} - \frac{x^7}{1.2.3.4.5.6.7} +$$

&c. — &c., the numerical value, or the material sine of x is readily determined.

82. Add together the logarithms of 1, 2, 3, 4, 5, 6, &c.

Log. 2 =	·301029995663981		
" 3 =	·477121254719662		
	·778151250383643	= log. 6	
" 4 =	·602059991327962		
	1·380211241711605	" 24	
" 5 =	·698970004336019		
	2·079181246047624	" 120	
" 6 =	·778151250383644		
	2·857332496431268	" 720	
" 7 =	·845098040014257		
	3·702430536445525	" 5040	
" 8 =	·903089986991044		
	4·605520523437469	" 40320	
" 9 =	·954242509439325		
	5·559763032876794	" 362880	
" 10 =	1·000000000000000		
	6·559763032876794	" 3628800	
" 11 =	1·041392685158225		
	7·601155718035019	" 39916800	
" 12 =	1·079181246047625		
	8·680336964082644	" 479001600	
" 13 =	1·113943352306837		
	9·794280316389481	" 6227020800	
" 14 =	1·146128035678238		
	10·940408352067719	" 87178291200	
" 15 =	1·176091259055681		

	12·116499611123400	"	1307674368000
" 16 =	1·204119982655925		
	13·320619593779325	"	20922789888000
" 17 =	1·230448921378274		
	14·551068515157599	"	355687428096000
" 18 =	1·255272505103306		
	15·806341020260905	"	6402373705728000

This process is continued beyond the required extent, but it may be useful in other investigations.

83. To find log. x.

4 0 9 4 4 3 9 2 7 7 6 7 4 3 5 (81.)
4 0 9 4 4 3 9 2 7 7 6 7 4 4

4 5|0 3|8 8|3 2|0 5|4 4|1 7|9 = A 1
1|3 5|1 1|6 4|9 6|1 6|3 2|5
1|3 5|1 1|6 4|9 6|1 6|3
4 5|0 3|8 8|3 2|1

4 6 4|0 3 5|5 3 7|0 4 4|9 8 8 = B 3
2|7 8 4|2 1 3|2 2 2|2 7 0
6|9 6 0|5 3 3|0 5 6
9|2 8 0|7 1 1
6|9 6 1
3

4 6 6 8|2 6 7 2|0 0 8 7|9 8 9 = C 6
9 3 3 6|5 3 4 4|0 1 8
4 6 6 8|2 6 7

4 6 6 9 2|0 0 9 0 1|0 0 2 7 4 = D 2
4 6 6 9 2|0 0 9 0 1

4 6 6 9 2 4 7 5 9 3 0 1 1 7 5 = E 1

This exceeds the constant.

4 6 6 9 2 4 6|8 3 2 8 7 7 7|5 8 .....
4 6 6 9 2 4 6|8

4 6 6 9 2 4 7 2|9|9|8|0|2|4|4 = G 1
2|8|0|1|5|4|8|4
1 = H 6
9|3|3|8|4|9 = I 2
3|2|6|8|4|7 = J 7
4|2|0|2|3 = K 9
2|3|3|5 = L 5
3|7|4 = M 8
1|4 = N 3
4 = O 8

4 6 6 9 2 4 7 5 9 3 0 1 1 7 5
The same result as at E 1.

·041392685158225 = 1 A
·012964121347929 = 3 B
..2604464875914 = 6 C
....86854553726 = 2 D
.....4342923104 = 1 E
—·057052468858898

Then
4 = 8 O
13 = 3 N
347 = 8 M
2172 = 5 L
39087 = 9 K
304010 = 7 J
868590 = 2 I
26057670 = 6 H
43429447 = 1 G

.4·669246832877758 add
4·669246903579098
—·057052468858898 subt.
4·612194434720200

∴ log. ·409443927767435=
log. x =
$\bar{1}$·6121944347202.

84. To find $\frac{x^3}{1\cdot2\cdot3}$.

Log. $x = \bar{1}\cdot612194434720200$

$\quad\quad\quad

To find $\frac{x^5}{1\cdot2\cdot3\cdot4\cdot5}$.

$\bar{1}\cdot6121944347202$
$\times\ 5$

$\bar{2}\cdot0609721736010$
$2\cdot0791812460476 = \log.\ (1\cdot2\cdot3\cdot4\cdot5)$

$\bar{5}\cdot9817909275534$ from this log. but 11 digits are required.

Const. $= 8\cdot95191599827$ subtract, neglecting the indices.

$\cdot02987492928$

6 B $= \cdot02592824270$ 8 9 | 5 1 | 9 1 | 5 9 | 9 8 | 2 7 Const.
$\cdot00394668658$ 8 9 | 5 1 | 9 1 | 5 9 | 9 8

9 C $= 390669731$ 9 0 | 4 1 | 4 3 | 5 1 | 5 8 | 2 5 $= B1$
$....3998927$ 4 | 5 2 | 0 7 | 1 7 | 5 7 | 9 1
9 E $= 3908631$ 9 | 0 4 | 1 4 | 3 5 | 1 6
$.....90296$ 9 | 0 4 | 1 4 | 3 5 $= B5$
1 F $= 86859$ 4 | 5 2 | 0 7
$.....3437$ 9 0
7 H $= 3040$ 9 5 0 | 2 6 3 | 9 2 1 | 8 6 4 $= B1+B5=B6$
$.....397$ 8 | 5 5 2 | 3 7 5 | 2 9 7
9 I $= 391$ 3 4 | 2 0 9 | 5 0 1
$.....6$ 7 9 | 8 2 2
1 J $= 4$ 1 2 0
$.....2$ 9 5 8 8 5 | 0 5 | 8 6 | 6 0 | 4 $= C9$
5 K $= 2$ 8 | 6 2 | 9 6 | 5 5 | 3
3 | 4 | 5 | 2 $= E9$
9 | 5 | 8 | 9 | 3 | 7 $= F1$
6 | 7 | 1 | 2 | 5 $= H7$
8 | 6 | 3 | 1 $= I9$
9 | 6 $= J1$
4 | 8 $= K5$

$\therefore\ \cdot0\ 0\ 0\ 0\ 9\ 5\ 8\ 9\ 3\ 7\ 9\ 2\ 1\ 4\ 4\ 6 =$ number sought.

85. To find $\frac{x^7}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7}$.

Log. $x = \bar{1}\cdot61219443472$
$\times\ 7$

log. $x^7 = \bar{3}\cdot28536104304$
$3\cdot70243053645 = \log.\ (1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7)$

$\bar{7}\cdot58293050659$ from this log. only 9 digits are required.

$\bar{7}\cdot58293050659$

Const. = $3\cdot55026018158$ ..... 3 5|5 0|2 6|0 1|8 2

$\cdot032670325$ 2|4 8|5 1|8 2|1 3

7B = $.30249616$ 7|4 5|5 5|4 6

$..2420709$ 1 2|4 2|5 9

5C = 2170387 1 2|4 3

$...250322$ 7

5D = 217136 3 8 0|6 3 5|9 4 5|0 = B 7

$....33186$ 1|9 0 3|1 7 9|7

7E = 30400 3|8 0 6|4

$.....2786$ 3|8|4

6F = 2606 3 8 2 5|4 2 9 3|4 9 = C 5

$.......180$ 1|9 1 2 7|1 5

4G = 174 3|8 3

$.........6$ 3 8 2 7 3|4|2|4|4|7 = D 5

1H = 4 2|6|7|9|1|4

$.........2$ 8 = E 7

5I = 2 2|2|9|6|6 = F 6

1|5|3|1 = G 4

3|8 = H 1

1|9 = I 5

$\cdot0\,0\,0\,0\,0\,0\,3\,8\,2\,7\,6\,3\,4\,9\,2\,3$

The index being 7, six ciphers must precede the number found.

86. To find the value of $\frac{x^9}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9}$.

$\bar{1}\cdot61219443472$

9

$\bar{4}\cdot50974991248$

$5\cdot55976303288 = \log.\,(1\cdot2\cdot2\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9) = \log.\,3628800\,(82).$

$10\cdot94998687960$ from this log. but 6 digits are required.

$7\cdot897489$ next less constant (6).

$\cdot052498$

$\cdot041393 = 1\,A$ 7 8 9 7 4 8 9

7 8 9 7 4 9

$\cdot011105$ 8 6|8 7|2 3|8 = A 1

$..8643 = 2\,B$ 1|7 3|7 4|5

$..2462$ 8 6|9

$2170 = 5\,C$ 8 8 6|1 8 5|2 = B 2

$...292$ 4|4 3 0|9

$261 = 6\,D$ 8|9

$....31$ 8 9 0 6|2|5|0 = C 5

$30 = 7\,E$ 5|3|4|4 = D 6

$.....1$ 6|2|3 = E 7

$1 = 2\,F$ 1|8 = F 2

The number required = $\cdot0\,0\,0\,0\,0\,0\,0\,0\,0\,8\,9\,1\,2\,2\,3\,5$

87. To find the value of $\frac{x^{11}}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10\cdot11}$.

Log. $x = \bar{1}\cdot61219443$
$\quad\quad\quad

3 9 4|4 3 3|9 0 6|1 7 4|2 7 0|3 = 1 B
3|5 4 9|9 0 5|1 5 5|5 6 8|4
1 4|1 9 9|6 2 0|6 2 2|3
3 3|1 3 2|4 4 8|1
4 9|6 9 8|7
4 9|7

3 9 7 9|9 8 0 4|4 1 3 2|6 5 7 5 = 9 C
7 9 5 9|9 6 0 8|8 2 6 5
3 9 7 9|9 8 0 4

3 9 8 0 7|7 6 4 7 7|2 1 4 6 4|4 = 2 D
1|9 9 0 3 8|8 2 3 8 6|1
3|9 8 0 7 7|6
4|0

3 9 8 0 9 7|5 5 2 0 0 1|9 3 2 1 = 5 E
1|5 9 2 3 9 0|2 0 8 0
2|3 8 8 6

3 9 8 0 9 9 1 4 4 3 9 4 5 2 8 7 = 4 F
1 1 9 4 2 9 7 4 3 3
1 1 9

3 9 8 0 9 9 2 6 3 8 2 4 2 8 3 9 = 3 G
1 5 9 2 3 9 7 0 5
2 = 4 H
7 9 6 1 9 8 6 = 2 I
3 1 8 4 7 9 4 = 8 J
1 1 9 4 = 3 M
1 9 9 = 5 N

3 9 8 0 9 9 2 8 0 8 6 3 0 7 1 9

Then the constant, 3·550260181586591

·041392685158225 = 1 A
·004321373782643 = 1 B
·003906697313871 = 9 C
....86854553726 = 2 D
21714615520 = 5 E
1737177060 = 4 F
130288341 = 3 G
17371780 = 4 H
868590 = 2 I
347440 = 8 J
130 = 3 M
22 = 5 N

3·599991393063939

∴ log. sin. (23° 27·5645405′) = 9·59991393063956.

. 4 1 3 9 2 6	9	1	3 7 1 2 8 8 6
. . 4 3 2 1 3	7	2	3 7 5 8 1 2 1
. . . 4 3 4 0	8	3	5 5 0 2 6 0 2
. . . . 4 3 4	3	4	6 6 9 2 4 6 8
. . . . . 4 3	4	5	7 6 0 4 5 6 9
. 4	3	6	8 3 4 7 2 0 8
. . . . .	4	7	8 9 7 4 8 9 0
. . . . .	.	8	9 5 1 9 1 6 0

88. Required the log. sine of 14° 21′.

	14° 21′	
	60	
m.....	86	1
a.....	1722	.
b.....	688	8
c.....	6	888
d.....		6888
e.....		689

·25045457 = the length of arc of 14° 21′.

89. This process is employed to multiply 861′ by ·000290888 :—

a is twice *m*, removed one figure to the left;
b is 4 times *a*, " " " right;
c is 4 times *a*, " three figures to the right;
d is the same as *c*, removed another figure to the right; and
e is found in like manner. The result is true to 8 places of decimals, which may be thus verified:

180° : 3·14159265 : : 14° 21′ : ·25045457.

To find the log. of ·25045457, use the 3d constant.

23758121	
1137906	
23758	
238	
1	
24970024	= B 5
74910	
75	= C 2
250	= E 1
175	= F 7
23	= G 9
25045457	

Again, 2·3758121

216069	= 5 B
13022	= 3 C
43	= 1 E
30	= 7 F
4	= 9 G
2·3987299	

∴ log. ·25045457 = $\bar{1}$·3987289.

To find $\frac{x^3}{1\cdot2\cdot3}$.

$\bar{1}$·3987289
3
$\bar{2}$·1961867
·7781513 = log. (1·2·3)
$\bar{3}$·4180354

From 2·4180354
Take 2·3758121
·04222
·04139 .. A
...83
43 .. C
...40
39 .. 9 D

2|3|7|5|8
2|3|7|6
2 6 1|3 4 .. A
2|6
2 6 1 6|0 .. C
2|4
2 6 1 8 4 .. D 9

∴ log. ·0026184 = $\bar{3}$·4180354.

To find $\frac{x^5}{1\cdot2\cdot3\cdot4\cdot5}$.

$\bar{1}$·3987289
5
$\bar{4}$·9936445
2·0791813 = log. $(\frac{x^5}{1\cdot2\cdot3\cdot4\cdot5})$
$\bar{6}$·9144632

From 7·9144632
Take 7·8974890
·017
·017 — 4 B

7 8|9 7
3|1 6
5
8 2 1 8 .. B 4

∴ log. ·0000082 = $\bar{6}$·9144632.

$$
\begin{aligned}
x &= \cdot 2504546 + \\
-\frac{x^3}{1\cdot 2\cdot 3} &= \cdot 0026184 - \\
\frac{x^5}{1\cdot 2\cdot 3\cdot 4\cdot 5} &= \cdot 0000082 + \\
\hline
\cdot 2478444 &= \text{Natural sine of } 14^\circ\ 21'.
\end{aligned}
$$

```
        2 3|7 5|8 1|2 1 Constant.
           |9 5|0 3|2 4
             1|4 2|5 5
                    9 5
      ------------------
B 4 — 2 4 7|2 2 7|9 5
           |4 9 4|4 5
                  |2 5
      ------------------
C 2 — 2 4 7 7|2 2 6 5
             |9 9 0 9 + 1
      ------------------
D 4 — 2 4 7 8 2|1 7 5
               2|2 3 1
      ------------------
E 9 — 2 4 7 8 4 4|0|6
                 |2|5 .. F
                  1|3 .. G 5
      ------------------
      2 4 7 8 4 4 4 4
```

```
2·3758121 Constant.
 ·0172855 .. 4 B
 ·0008682 .. 2 C
  ...1737 .. 4 D
  ....391 .. 9 E
  ......4 ..   F
  ......2 .. 5 G
-----------
2·3941792
```

$\therefore$ log. $\cdot 2478444 = \bar{1}\cdot 3941792$, and log. sin. $14^\circ\ 21' = 9\cdot 3941792$.

90. Required the log. cosine of $45^\circ\ 56\cdot 091'$.

```
   45° 56·091′
    60
  ----------
  2756·091
 ·55121·82
 22048·728
   220·487
    22·049
     2·205
 ----------
 ·80171380; put = x. [24]
```

It is well known that

$$\text{cosine } x = 1 - \frac{x^2}{1\cdot 2} + \frac{x^4}{1\cdot 2\cdot 3\cdot 4} - \frac{x^6}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6} + \text{\&c.}$$

$$
\begin{aligned}
& & 1\cdot 0000000 \\
-\frac{x^2}{1\cdot 2} &= & \cdot 3213725 \\
\hline
& & \cdot 6786275 \\
+\frac{x^4}{1\cdot 2\cdot 3\cdot 4} &= & \cdot 0172132 \\
\hline
& & \cdot 6958407
\end{aligned}
$$

$$-\frac{x^6}{1\cdot2\cdot3\cdot4\cdot5\cdot6} = \begin{array}{r}\cdot6958407\\ 0003668\\ \hline \cdot6954719\end{array}$$

$$+\frac{x^8}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8} = \begin{array}{r} 0000042\\ \hline \cdot6954761\end{array}$$

These results may be obtained by common arithmetic without much labor.

6	8	3	4	7	2	0	8	Constant.	
	6	8	3	4	7	2			
6	9	0	3	0	6	8	0	.. B	
	4	8	3	2	1	4			
		1	4	5	0				
6	9	5	1	5	3	4	4	.. C	7
		2	7	8	0	6			
						4			
6	9	5	4	3	1	5	4	.. D	4
			4	1	7	2		.. E	6
				2	7	8		.. F	4
6	9	5	4	7	6	0	4		

Constant.		
6·8347208		
..43214 ..		B
..20385 ..	7	C
...1737 ..	4	D
....261 ..	6	E
.....17 ..	4	F
6·8422822		

Hence log. 6954761· $= 6\cdot8422822$

$\therefore$ log. $\cdot6954761 = \bar{1}\cdot8422822$

$\therefore$ log. cos. 45° 56·091′ $= 9\cdot8422822$

91. What is the log. sine of 1° 12·5721′?

1° 12·5721′
60
72·5721
·01451 442
14 5144
1 4514
1451

·015401250 = length of arc of 1° 12·5721′ radius = 1.

Log. ·015401250 $= \bar{2}\cdot1875559$
3
$\bar{6}\cdot5626677$
·7781513 = log (1·2·3)
log. ·000000608 $= \bar{7}\cdot7845164$

$x = \cdot015401250$

$$-\frac{x^3}{1\cdot2\cdot3} = \cdot000000608$$

Natural sine 1° 12·5721′ = ·015400642

$\bar{2}\cdot1875388$ = log. ·015400642, hence log. sin. 1° 12·5721′
= 8·1875388.

92. What is the value of $(3\cdot1416)^{\frac{3}{5}}$ to five places of figures?

0 4 1 3 9	3
. . 4 3 2	1
. . . 4 3	4
. . . . 4	3
. . . . .	4

1	3 7 1 2 9
2	3 7 5 8 1
3	5 5 0 2 6
4	6 6 9 2 5
5	7 6 0 4 6
6	8 3 4 7 2
7	8 9 7 4 9
8	9 5 1 9 2

	3 1 4 1 6 0	
	3 1 4 1 6	
A —	3 4 5 5 7 6	—
	6 9 1 1	
	3 5	
B 2 —	3 5 2 5 2 2	
	2 4 6 8	+ 7
C 7 —	3 5 4 9 9 7	
	2 8	
E 8 —	3 5 5 0 2 5	

	·04139 .. A
	·00864 .. 2 B
	·00304 .. 7 C
	·00003 .. 8 E
Take	·05310
From	3·55025
	3·49715 log. 3141·6

∴ ·49715 = log. 3·1416.

	·49715	
	3	
	5)1·49145	
	·29829	
Constant	·13713	
	·16116	
	12418	.. 3 A
	03698	
	03457	.. 8 B
	00241	
	..217	.. 5 C
	00024	
	...25	.. 6 D

	1 3 7 1 3	
	4 1 1 4	
	4 1 1	
	1 4	
A 3 —	1 8 2 5 2	
	1 4 6 0	
	5 1	+ 1
B 8 —	1 9 7 6 4	
	9 9	
C 5 —	1 9 8 6 3	
	1 2	
D 6 —	1 9 8 7 5	

Hence the cube of the fifth root of 3·1416 is 1·9875, true to five places of figures.

What will $1 amount to in 23 years, at 6 per cent., compound interest?

1	0	6	0	0	0	0		
	2	1	2	0	0	0		
		1	0	6	0	0		
1	2	8	2	6	0	0	..	A 2
		7	6	9	5	6		
			1	9	2	4		
					2	6		
1	3	6	1	5	0	6	..	B 6
			9	5	3	1		
					2	9		
1	3	7	1	0	6	6	..	C 7
			1	3	7	..	D	
				8	2	..	E 6	
					3	..	F 2	
1	3	7	1	2	8	8		

		3	5	5	0	2	6	0
			2	4	8	5	1	8
					7	4	5	6
						1	2	4 + 1
B 7	—	3	8	0	6	3	5	9
				1	1	4	1	9
							1	1
C 3	—	3	8	1	7	7	8	9
					1	9	0	9
								4
D 5	—	3	8	1	9	7	0	2
							7	6
E 2	—	3	8	1	9	7	7	8

·082785	..	2	A
.25928	..	6	B
..3039	..	7	C
....43	..		D
....26	..	6	E
.....1	..	2	F
·111822	take		
·137128	from		
·025306			
23			
·075918			
50612			
·582038			
3·550260	Constant.		
·031778			
·030250	..	7	B
·001528			
..1302	..	3	C
·000226			
217	..	5	D
·000009			
·000009	..	2	E

∴ $1 amounts to $3·82 in 23 years, at 6 per cent., compound interest.

The method of finding the constants (6) was first given in my work on the Calculus; see *O. Byrne's Calculus* (458), from which the following is taken.

93. Required the number whose common logarithm and itself are each composed of the same digits.

The equation to be solved is $10^{\frac{+1}{10}} = 1 + z$, find z.

$z = (-1) + 10^{\frac{1}{10}} 10^{\frac{z}{10}}$; $y = (-1)$, $z = 10^{\frac{1}{10}}$, for, $z = y + xf(z)$; generally (450) by Lagrange's theorem, putting F instead of Ψ,

$$u = F(z) = F(y + xf(z)) = F(y) + \frac{d \cdot F(y)}{dy} f(y) \frac{x}{1} +$$

$$\frac{d\cdot\left\{\frac{d\cdot F(y)}{dy}(f(y))^2\right\}}{dy}\ \frac{x^2}{1\cdot 2}+\frac{d.^2\left\{\frac{d\cdot F(y)}{dy}(f(y))^3\right\}}{dy^2}\ \frac{x^3}{1\cdot 2\cdot 3}+\text{\&c.}$$

We have to determine the simplest possible function of z, namely z itself. $\therefore u = F(z) = z$, hence the nature of the function expressed by F becomes known; in this example, therefore, $F(y) = y$. And $f(z) = 10^{\frac{z}{10}}$, hence, also, the nature of the function expressed by f becomes known, $\therefore f(y) = 10^{\frac{y}{10}}$.

$$F(y) = y = -1 \qquad \text{(A)}.$$

$$\frac{d\cdot F(y)}{dy}f(y)\frac{x}{1} = \frac{d\cdot y}{dy}f(y)\ \frac{x}{1} = \frac{dy}{dy}\,10^{\frac{y}{10}}\,10^{\frac{1}{10}} = 10^{\frac{-1}{10}}\,10^{\frac{1}{10}} = +1 \text{ (B)}.$$

$$\frac{d.\left\{\frac{d\cdot F(y)}{dy}(f(y))^2\right\}}{dy}\ \frac{x^2}{1\cdot 2} = \frac{d\cdot\{(f(y))^2\}}{dy}\ \frac{x^2}{1\cdot 2} = \frac{d\cdot 10^{\frac{2y}{10}}}{dy}\ \frac{x^2}{1\cdot 2} =$$

$$log.\,10\,10^{\frac{2y}{10}}\,\frac{2}{10}\,\frac{x^2}{1\cdot 2} = \frac{2}{10}\,(log.\,10)\,10^{\frac{2y}{10}}\,\frac{10^{\frac{2}{10}}}{1\cdot 2} =$$

$\frac{2}{10}\,(log.\,10)\,\frac{1}{1\cdot 2}$ when the function of y is substituted. (C).

$$\frac{d.^2\left\{\frac{d\cdot F(y)}{dy}\,f(y))^3\right\}}{dy^2}\ \frac{x^3}{1\cdot 2\cdot 3} = \frac{d^2.(f(y))^3}{dy^2}\ \frac{x^3}{1\cdot 2\cdot 3} =$$

$$\left(\frac{3}{10}\right)^2\,(log.\,10)^2\,\frac{1}{1\cdot 2\cdot 3} \qquad \text{(D)}.$$

$$\frac{d.^3\left\{\frac{d\cdot F(y)}{dy}(f(y))^4\right\}}{dy^2}\ \frac{x^4}{1\cdot 2\cdot 3\cdot 4} = \frac{d.^3(f(y))^4}{dy^3}\ \frac{x^4}{1\cdot 2\cdot 3\cdot 4} =$$

$$\left(\frac{4}{10}\right)^3\,(log.\,10)^3\,\frac{1}{1\cdot 2\cdot 3\cdot 4} \qquad \text{(E)}.$$

$$\frac{d.^4\left\{\frac{d\cdot F(y)}{dy}\,(f(y))^5\right\}}{dy^4}\ \frac{x^5}{1\cdot 2\cdot 3\cdot 4\cdot 5} = \frac{d.^4\,(f(y))^5}{dy^4}\ \frac{x^5}{1\cdot 2\cdot 3\cdot 4\cdot 5} =$$

$$\left(\frac{5}{10}\right)^4\,(log.\,10)^4\,\frac{1}{1\cdot 2\cdot 3\cdot 4\cdot 5} \qquad \text{(F)}.$$

$$\therefore u = F(z) = z = \text{A} + \text{B} + \text{C} + \text{D} + \text{E} + \text{F} + \text{\&c.}$$

$$\therefore z = \left(\frac{2}{10}\right)^1 (log.\ 10)\ \frac{1}{1\cdot 2} + \left(\frac{3}{10}\right)^2 (log.\ 10)^2 \frac{1}{1\cdot 2\cdot 3} + \left(\frac{4}{10}\right)^3 (log.\ 10)^3 \frac{1}{1\cdot 2\cdot 3\cdot 4} + \&c.$$

$= 371288574238542$. Hence $log.\ 1\cdot 371288574238542 = \cdot 137288574238542$.

In a similar manner the other constants of (6) may be determined.

To verify 1371288574238542.

$$\frac{37}{27} = 1\cdot 3\dot{7}\ 0\ \begin{array}{|c|c|c

$$
\begin{array}{rl}
\ldots\ldots.017371780 & = \text{H } 4 \\
\ldots\ldots\ldots.02605770 & = \text{I } 6 \\
\ldots\ldots\ldots.0217150 & = \text{J } 5 \\
\ldots\ldots\ldots.034744 & = \text{K } 8 \\
\ldots\ldots\ldots.02606 & = \text{L } 6 \\
\ldots\ldots\ldots.0043 & = \text{M } 1 \\
\ldots\ldots\ldots.22 & = \text{N } 5 \\
\ldots\ldots\ldots.04 & = \text{O } 8 \\
\hline
\cdot 1371288574238900 & = \text{the logarithm of} \\
1\cdot 371288574238655. &
\end{array}
$$

Here the digits of the number and its log. are not alike in the 14th and 15th decimal places. But the difference of the consecutive logs. of numbers beginning with 1371288, &c., when carried to 15 places of decimals are equivalent to

$$\frac{8685889638065040}{1371288574238655 \times 2 + 1} = 3\cdot 17;$$

according to a well known property of logarithms.

Let $x =$ the number which taken from the given number will render the digits which compose it the same as its log.

$$
\begin{array}{rl}
\text{Then } 655 - x & = 900 - 3\cdot 17x; \\
2\cdot 17x & = 245 \\
\therefore x & = 113; \text{ and } 3\cdot 17x = 358 \text{ very nearly.}
\end{array}
$$

Hence 1371288574238900 — 358 = 1371288574238655 — 113, or
Log. 1·371288574238542 = 1371288574238542.

To verify the second constant.

$$
\begin{array}{rl}
2\,3\,7\,5\,|0\,0\,0\,0\,|0\,0\,0\,0\,|0\,0\,0\,0 & \\
\quad\;|7\,1\,2\,5\,|0\,0\,0\,0\,|0\,0\,0\,0 & \\
\quad\;\;\;|7\,1\,2\,5\,|0\,0\,0\,0 & \\
\quad\;\;\;\;\;|2\,3\,7\,5 & \\
\hline
\ldots\ldots 7\,|1\,2\,5\,7\,1\,|2\,5\,2\,3\,7\,|5 & = \text{D } 3 \\
\quad\;|9\,5\,0\,2\,8\,|5\,0\,2\,8\,5\,|0 & \\
\quad\;\;\;\;\;1\,|4\,2\,5\,4\,2\,|8 & \\
\quad\;\;\;\;\;

$$\begin{array}{rrrr

```
. . . . . 5 9|1 5 8 2 1 1 2|9 7 = F 3
          |7 1 0 0 5 1 8|3 2
                        3 6
          -------------------
. . . . . 5 9 8|6 8 2 6 3 1 6 5 = G 2
            2|8 4 0 2 0 7 8 9
                          1 0
          -------------------
. . . . . 6 0 1 5|2|2|8|3 9 6 4 = H 8
            2|8|4|0|2 0 8 1 = I 8
              |7|1|0 0 5 2 = J 2
              1|7|7 5 1 3 = K 5
                1|4 2 0 1 = L 4
-----------------------------------
3 5 5 0 2 6 0 1 8 1 5 8 7 8 1 1
```

```
Log. 71 = 1·851258348719075
log. 50 = 1·698970004336019
          000030400461728 = 7 E
          ....01302882795 = 3 F
          .....0086858894 = 2 G
          .......34743560 = 8 H
          .......03474360 = 8 I
          ........0086860 = 2 J
          .........21715 = 5 K
          .........01734 = 4 L
          ------------------
          3·550260181586740
```

In this case the constant is greater than its logarithm.

$\frac{868588963806504}{3550260181587811 \times 2 + 1} = \cdot 1223$ the difference of the consecutive logarithms.

Hence we have $7811 - x = 6740 - \cdot 1223\,x$;

$\therefore \cdot 8777\,x = 1071$, and $x = 1220$.

```
From 3550·260181587811
Take             1220
-----------------------
  3550·260181586591  the number sought.
```

To verify the fourth constant.

```
4 6 6 5|6·0 0 0|0 0 0 0|0 0 0 0 = (36)³
     3|2 6 5 9|2 0 0 0|0 0 0 0
          9|7 9 7 7|6 0 0 0
              1 6|3 2 9 6
                    1 6
-----------------------------
```

```
. . . 8 8|6 6 8 9 9|9 3 9 3 1|2 = D7
      3|7 3 5 0 9|3 5 1 9 9|5
             1 3|0 7 2 8 2|7
                      2 6|1
. . . 9 2 4|0 4 2 2 3 6|4 3 9 5 = E8
           |4 6 6 9 2 4|0 4 2 2
. . . . . . 5|0 9 1 6 0 4 8|1 7 = F1
           1|4 0 0 7 7 3 5|2 7
                        1|4 0
. . . . . . 6 4|9 2 3 7 8 4 8 4 = G3
             3|2 6 8 4 7 2 5 4
                          1 0
. . . . . . . 8 1|9|2|2|5|7 4 8 = H7.
               |9|3|3|8|4 9 4 = I2
               4|2|0|2|3 2 2 = J9
                  9|3|3 8 5 = K2
                  1|4|0 0 8 = L3
                   3|7 3 5 = M8
4 6 6 9 2·4 6 8 3 2 8 7 7 6 9 2
```

Log. 36 = 1·556302500767287, 265
3

log. 46656 = 4·668907502301862

·000303992938041 = 7 D
. . .034753384832 = 8 E
. . . .00434294265 = 2 F
. . . . . .130288341 = 3 G
. . . . . .030400615 = 7 H
. . . . . .00868590 = 2 I
. . . . . .390870 = 9 J
. . . . . .008686 = 2 K
. . . . . .1303 = 3 L
. . . . . .0347 = 8 M

4·669246832877752 the logarithm of 46692.46832877692. The difference between the logarithms of 46692·468328776, 46692·468428777, 46692·468328778, &c., respectively, is equivalent to

$$\frac{\cdot 868588963806504}{9338493665755385}, \quad \frac{\cdot 868588963806504}{9338493665755387}, \quad \frac{\cdot 868588963806504}{9338493665755388},$$

&c. = ·093. Let x = the number that will make this constant and its logarithm consist of the same digits.

Then, $692 + x = 752 + \cdot 093\,x$; $\therefore\ x - \cdot 093\,x = 752 - 692 = 60$; and $\cdot 907\,x = 60$; $\therefore\ x = \frac{60000}{907} = 66$.

To 46692·46832877692
Add 66

46692·46832877758 a number, the logarithm of which is expressed by the same figures.

To verify the fifth constant.

$2400 \times 240 = 576000|000000|000000|0$
$4|032000|000000|0$
$12|096000|0$
$20|2$

$\ldots\ldots 40|321209|6202 = E7$
$5|184362|8909$
$20|7374$

$\ldots\ldots 55|055932|485 = F9$
$1|728136|517$
$1|73$

$\ldots\ldots 67|840691|75 = G3$
$1|352091|36$
1

$\ldots\ldots 91|927831|2 = H2$
$3|456274|1$

$\ldots\ldots 538|41|0|53 = I6$
$2|8|8|0|23 = K5$
$5|7|60 = L1$
$5|76 = M1$
$1|15 = N2$

$576045{\cdot}695413552 7$

Log. 240 = 2·380211241711606
log. 2400 = 3·380211241711606

000030400461728 = 7E
....03908648385 = 9F
.....0130288341 = 3G
......008685890 = 2H
.........2605770 = 6I
.........0021715 = 5K
..........00434 = 1L
............043 = 1M
.............09 = 2N

5·760456934135527

This exactly coincides with the number, hence it is unnecessary to emyloy the usual equation.

We shall in the next place test the accuracy of the *constant* beginning with 6834.

6 8 3 4 | 0 0 0 0 | 0 0 0 0 | 0 0 0 0 = 2010 × 3400
6 8 3 4 | 0 0 0 0 | 0 0 0 0

. . . . 6 8 | 3 4 0 0 0 0 | 0 0 0 0 = D 1
3 | 4 1 7 3 4 1 | 7 0 0 0
6 | 8 3 4 7
1

. . . . 7 1 7 | 5 7 3 4 8 5 3 | 4 8 = F 5
2 | 7 3 3 8 8 7 0 | 2 9
4 | 1 0

. . . . . 2 0 3 | 0 7 3 7 2 7 8 7 = G 4
4 | 1 0 0 8 3 2 1 8
1 0

. . . . . 7 1 | 7 | 4 | 5 | 6 0 | 1 | 5 = H 6
5 | 4 | 6 | 7 | 7 7 | 6 | 6 = I 8
4 | 1 | 0 | 0 8 | 3 | 6 = J 6
4 | 7 | 8 4 | 3 | 0 = K 7
4 | 1 0 | 0 | 8 = L 6
3 | 4 | 2 = N 5
7 = O 1

6 8 3 4 7 2 0 · 7 7 6 7 5 4 3 9 8

Log. 2010 = 3·303196057420489
log. 3400 = 3·531478917042255
000043427276863 = 1 D
....02171471325 = 5 F
.....0173717788 = 4 G
.....026057670 = 6 H
.....03474360 = 8 I
.....0260580 = 6 J
.....030401 = 7 K
.....02606 = 6 L
.....0022 = 5 N
.....01 = 1 O

6·834720776754360 the logarithm of 6834720·776754398 agreeing to the 15th digit.

The difference of the consecutive logs. in this instance =

$$\frac{868588963806504}{6834720776754398 \times 2 + 1} = \cdot 06354.$$

Hence we have the three last figures of the number $398 - x =$ the three last figures of the logarithm, $360 - \cdot 06354\, x$;

$$\therefore\ 38 = x - \cdot 06354\, x = \cdot 93646\, x;$$

$$\therefore x = \frac{38}{\cdot 93646} = 41 \text{ nearly.}$$

Then from 6834720·776754398
take 41

6834720·776754357 = the required number.

To verify the correctness of the number commencing with 7897.

7 8 9 6|0 0 0 0|0 0 0 0|0 0 0 0 = 9400 × 8400
7 8 9 6|0 0 0 0|0 0 0 0

. . . 6 7|8 9 6 0 0|0 0 0 0 0|0 = D 1
6|3 1 7 4 3|1 6 8 0 0|0
2 2|1 1 1 0 1|1
4 4|2

. . . 7 4 2|1 3 6 5 2 7|9 4 5 3 = E 8
6|3 1 7 9 3 7|0 9 2 2
2 2|1 1 2 7
1

. . . . . 8 4|5 4 4 8 7 1 5|0 3 = F 8
3|9 4 8 7 4 2 2|7 2
7|9 0

. . . . . 8 4|9 3 6 1 4 5 6 5 = G 5
4|7 3 8 4 9 3 1 0
1 2

. . . . . 8 9 6|7|4|6|3|8|8 7 = H 6
6|3|1|7|9|9|1 1 = I 8
7|1|0|7|7 4 = K 9
3|9|4|8 7 = L 5
3|9|4 9 = M 5
|7 9 = N 1
|5 5 = O 7

7 8 9 7 4 8 9 0·3 1 3 9 8 1 4 2

Log. 9400 = 3·973127853599699
log. 8400 = 3·924279286061882
000043427276863 = 1 D
....34743384832 = 8 E
....03474354120 = 8 F
.....0217147235 = 5 G
......026057670 = 6 H
.......03474360 = 8 I
........0039087 = 9 K
.........02172 = 5 L
..........0217 = 5 M
.........004 = 1 N
.........3 = 7 O

7·897489031398144

$$\frac{868588963806504}{7897489031398142 \times 2 + 1} = \cdot 055$$ the difference of the consecutive logarithms.

Consequently $2 + x = 4 + \cdot 055\, x$;
$\therefore \cdot 945\, x = 2$;
and $x = 2$ very nearly.

Hence the logarithm of 78974890·31398144 is expressed by the same digits; for ·055 added to the logarithm will not increase it in the 15th decimal place.

We have but one more constant to verify; that is the one beginning with 895191.

```
8 9 5 0|0 0 0 0|0 0 0 0|0 0 0 0 = 179 × 50.
     1|7 9 0 0|0 0 0 0|0 0 0 0
              |8 9 5 0|0 0 0 0
  . . . 1 7|9 0 0 8 9|5 0 0 0 0|0 = D 2
           |8 9 5 1 7|9 0 0 8 9|5
  . . . . 8 7|9 6 0 7 4 0|0 8 9 5 = E 1
           3|5 8 0 7 5 1|8 4 3 0
                      5|3 7 1 1
  . . . . 9 1 5 4|1 4 9 7 3 0 3 6 = F 4
                5|3 7 1 1 4 9 2 5
                               1 3
  . . . . . . . 9 5|2|0|8|7|9|7|4 = H 6
                  4|4|7|5|9|5|8|0 = I 5
                  0|0|8|9|5|1|9|2 = J 1
                     4|4|7|5|9|6 = K 5
                     0|7|1|6|1|5 = L 8
                       0|5|3|7|1 = M 6
                         0|1|7|9 = N 2
                           0|6|3 = O 7
8 9 5 1 9 1 5 9 9 8 2 6 7 5 7 0
```

```
Log. 50000 = 4·698970004336019
 log. 17900 = 4·252853030979893
              ·000086854553726 = 2 D
               ....04342923104 = 1 E
               .....1737177060 = 4 E
               .....0026057670 = 6 H
               .......02171475 = 5 I
               ........0043430 = 1 J
```

$$\begin{array}{rl} \ldots\ldots\ldots\ldots 21715 & = 5\,\mathrm{K} \\ \ldots\ldots\ldots\ldots 03474 & = 8\,\mathrm{L} \\ \ldots\ldots\ldots\ldots 0261 & = 6\,\mathrm{M} \\ \ldots\ldots\ldots\ldots 009 & = 2\,\mathrm{N} \\ \ldots\ldots\ldots\ldots 3 & = 7\,\mathrm{O} \\ \hline 8{\cdot}951915998267839 & \end{array}$$

this logarithm is more than its corresponding number by 269.

$$\frac{868588963806504}{8951915998267570 \times 2 + 1} = {\cdot}0485 \text{ the difference.}$$

$$\begin{aligned} \text{Then we have } 570 + x &= 839 + {\cdot}0485\,x\,; \\ \therefore\ {\cdot}9515\,x &= 269\,; \\ \text{and } x &= 282. \end{aligned}$$

$\therefore$ $8951915998267570 + 282 = 8951915998267839 + {\cdot}0485\,x =$ $895191599{\cdot}8267852$, a number whose logarithm is expressed by the same digits.

I shall conclude with a TABLE of the logarithms of all the integer numbers under 221, to fifty places, more than one-half of which have not been before calculated to any thing like this extent.

N.	Logarithms to 50 Decimal Places
1	0·00000000000000000000000000000000000000000000000000
2	0·30102999566398119521373889472449302676818988146211
3	0·47712125471966243729502790325511530920012886419069
4	0·60205999132796239042747778944898605353637976292422
5	0·69897000433601880478626110527550697323181011853789
6	0·77815125038364363250876679797960833596831874565280
7	0·84509804001425683071221625859263619348357239632397
8	0·90308998699194358564121668417347908030456964438633
9	0·95424250943932487459005580651023061840025772838139
10	1·00000000000000000000000000000000000000000000000000
11	1·04139268515822504075019997124302424170670219046645
12	1·07918124604762482772250569270410136273650862711491
13	1·11394335230683676920650515794232843082972918838707
14	1·14612803567823802592595515331712922025176227778607
15	1·17609125905568124208128900853062228243193898272859
16	1·20411998265592478085495557889797210707275952584843
17	1·23044892137827392854016989432833703000756737842505
18	1·25527250510330606980379470123472364516844760984350
19	1·27875360095282896153633475756929317951129337394 50
20	1·30102999566398119521373889472449302676818988146211
21	1·32221929473391926800724416184775150268370126051866
22	1·34242268082220623596393886596751726847489207192856
23	1·36172783601759287886777711225118954969751103433610
24	1·38021124171160602293624458742859438950469850857702
25	1·39794000867203760957252221055101394646362023707578
26	1·41497334797081796442024405266682145759791906984918
27	1·43136376415898731188508370976534592760038659257209
28	1·44715803134221922113969404804162224701995215924818
29	1·46239799789895608733284676296925499125429441788715
30	1·47712125471966243729502790325511530920012886419070
31	1·49136169383427267966670410011841572230370155830418
32	1·50514997831990597606869447362246513384094940731054
33	1·51851393987788747804522787449813955090683105465715
34	1·53147891704225512375390878905283005677575725988715
35	1·54406804435027563549847736386814316671538251486186
36	1·55630250076728726501753359595921667193663749130561
37	1·56820172406699499680845068953912944798297269016631
38	1·57978359661681015675007237048142234471931921885661
39	1·59106460702649920650153306119744374002985805257776
40	1·60205999132796239042747778944898605353637976292422

N.	Logarithms to 50 Decimal Places.
41	1·61278384671973549450941184996818079953051363383369
42	1·62324929039790046322098305657224452945189114197677
43	1·63346845557958652640508815322922215880877488438009
44	1·64345267648618743117767776069201029524308195339067
45	1·65321251377534367937631691178573759163206784691928
46	1·66275783168157407408151600697568257646570091579820
47	1·67209785793571746441421939944920064015980309842995
48	1·68124123737558721814998348215308741627288839003913
49	1·69019608002851366142443251718527238696714479264793
50	1·69897000433601880478626110527550697323181011853789
51	1·70757017609793636583519779758345233920769624261574
52	1·71600324363479915963398294739131448436610895131129
53	1·72427586960078904563299229162725659269550240129494
54	1·73239375982296850709882260448983895436857647403420
55	1·74036268949424384553646107651853121493851230900434
56	1·74818802700620041635343294276611527378814204071029
57	1·75587485567249139883136137901204462715125820158519
58	1·76342799356293728254758565769374801802248429934926
59	1·77085201164214419026065638453514423892674447493077
60	1·77815125038364363250876679797960833596831874565280
61	1·78532983501076703388874851375732134926337875711340
62	1·79239168949825387488044299484290874907189143976629
63	1·79934054945358170530227206510286681188383012470536
64	1·80617997398388717128243336834695816060913928877265
65	1·81291335664285557399276626321783540406153930692496
66	1·81954393554186867325896676922263257767502093611926
67	1·82607480270082643414913162922606858094962608056861
68	1·83250891270623631896764768377732308354394714134926
69	1·83884909073725531616280501550630485889763989852679
70	1·84509804001425683071221625859263619348357239632397
71	1·85125834871907528609282943503542913527041990160039
72	1·85733249643126846023127249068370969870482737276772
73	1·86332286012045590107438690047030853445286825531166
74	1·86923171973097619202218958426362247475116257162842
75	1·87506126339170004686755011380612925566374910126648
76	1·88081359228079135196381126520591537148750910031871
77	1·88649072517248187146241622983566043519027458679042
78	1·89209460269048040171527195592193676679804793403987
79	1·89762709129044142799482138647824968648286201902515
80	1·90308998699194358564121668417347908030456964438633

N.	Logarithms to 50 Decimal Places.
81	1·90848501887864974918011161302046123680051545676278
82	1·91381385238371668972315074469267382629870351529580
83	1·91907809237607390383276035202726124700163765808063
84	1·92427928606188165843472195129673755622008102343888
85	1·92941892571429273332643099960384400323937749696294
86	1·93449845124356772161882704795371518557696476584220
87	1·93951925261861852462787466622437030045442328207785
88	1·94448267215016862639141665541650332201127183185278
89	1·94939000664491278472354336970244112466516185810024
90	1·95424250943932487459005580651023061840025772838139
91	1·95904139232109359991872141653496462431330158471103
92	1·96378782734555526929525490170017560323389079726031
93	1·96848294855393511696173200337353103150383042249488
94	1·97312785359969865962795829417369366692799297989206
95	1·97772360528884776632259458103243629118293945593239
96	1·98227123303956841336372237687758044304107827150124
97	1·98677173426624485178436181166557744942584158463887
98	1·99122607569249485663817141190976541373533467411004
99	1·99563519459754991534025577775325486010695991884784
100	2·00000000000000000000000000000000000000000000000000
101	2·00432137378264257427518817822293791321928935520645
102	2·00860017176191756104893669230794536597588612407785
103	2·01283722470517220517107119458023942439052349697603
104	2·01703333929878035484772184211580751113429883277339
105	2·02118929906993807279350526712325847591551137905655
106	2·02530586526477024084673118635174961946369228275705
107	2·02938377768520964083454123946143564612681689163402
108	2·03342375548694970231256149921433198113676635549630
109	2·03742649794062363520051330761387528664220452282798
110	2·04139268515822504075019997124302424170670219046645
111	2·04532297878665743410347859279424475718310155435701
112	2·04921802267018161156717183749060830055623192217240
113	2·05307844348341972279522702860944818477838362362210
114	2·05690485133647259404510027373653765391944808304730
115	2·06069784035361168365403821752669652292932115287399
116	2·06445798922691847776132455241824104479067418081137
117	2·06818586174616164379656096445255904922998691676846
118	2·07188200730612538547439527925963726569493435639287
119	2·07554696139253075925238615292097322349113977474901
120	2·07918124604762482772250569270410136273650862711491

N.	Logarithms to 50 Decimal Places.
121	2·08278537031645008150039994248604848341340438093291
122	2·08635982667474822910248740848181437603156863857551
123	2·08990510143939793180443975322329610873064249802438
124	2·09342168516223507009418188956740177584008132122840
125	2·09691001300805641435878331582652091969543035561367
126	2·10037054511756290051601095982735983865202000616747
127	2·10380372095595686424698742184728625857656323979239
128	2·10720996964786836649617226307145118737732917023476
129	2·11058971029924896370011605648433746800890374857079
130	2·11394335230683676920650515794232843082972918838707
131	2·11727129565576420298526262090351324991078785674493
132	2·12057393120584986847270566394712560444321081758137
133	2·12385164096708579224854973434956551143470173371846
134	2·12710479836480762936287052395056160771781596203072
135	2·13033376849500611667134481504085290083219671110998
136	2·13353890837021751418138657850181611031213702281137
137	2·13672056715640676856292662711478973367822970746424
138	2·13987908640123651137654391023079788566582977998890
139	2·14301480025409508045643320231984731447973296791786
140	2·14612803567823802592595515331712922025176227778607
141	2·14921911265537990170924730270431594935993196262064
142	2·15228834438305648130656832975992216203860978306250
143	2·15533603746506180995670512918535267253643137885352
144	2·15836249209524965544501138540820272547301725422983
145	2·16136800223497489211910786824476196448610453642505
146	2·16435285578443709628812579519480156122105813677377
147	2·16731733474817609871946042044038769616727365684263
148	2·17026171539495738723592847898811550151935245309053
149	2·17318626841227403825736354262833705393467132637222
150	2·17609125905568124208128900853062228243193898272859
151	2·17897694729316943686907305533730278844609342877688
152	2·18184358794477254717755015993040839825569898178082
153	2·18469143081759880313022570083856764840782510680644
154	2·18752072083646306667615512456015346195846446825253
155	2·19033169817029048445296520539392269553551167684208
156	2·19312459835446159692901085064642979356623781550198
157	2·19589965340923373676148112989728370506519099278553
158	2·19865708695442262320856028120274271325105190048726
159	2·20139712432045148292802019488237190189563126548563
160	2·20411998265592478085495557889797210707275952584843

N.	Logarithms to 50 Decimal Places.
161	2·20682587603184970957999337084382574318108343066006
162	2·20951501454263094439385050774495426356870533822589
163	2·21218760440395780764226290833551101413135289195443
164	2·21484383804769788493688963941716685306689339675790
165	2·21748394421390628283148897977364652413864117319504
166	2·22010808804005509904649924675175427376982753954274
167	2·22271647114758327998407590992046753446133840133780
168	2·22530928172586285364846084602123058298827090490099
169	2·22788670461367353841301031588465686165945837677414
170	2·23044892137827392854016989432833703000756737842505
171	2·23299611039215383612638928226715993635138706577589
172	2·23552844690754891633256594267820821234515464730431
173	2·23804610312882201456053025875846588778168326913493
174	2·24054924828259971984261356094886332722261316353996
175	2·24303804868629444028473846914365013994719273339975
176	2·24551266781414982160515555014099634877946171331489
177	2·24797326636180662755568428779025954812687333912146
178	2·25042000230889397993728226442693415143335173956235
179	2·25285303097989316957038269177305861943107209067853
180	2·25527250510330606980379470123472364516844760984350
181	2·25767857486918451028974367611229249224795923272292
182	2·26007138798507479513246031125945765108149146617314
183	2·26245108973042947118377641701243665846350762130410
184	2·26481782300953646450899379642466863000208067872242
185	2·26717172840301380159471179481463642121478280870420
186	2·26951294421791631217547089809802405827202030395699
187	2·27184160653649896929036986557136127171426956889150
188	2·27415784926367985484169718889818669369618286135416
189	2·27646180417324314259729996835798212108395898889605
190	2·27875360095282896153633347575692931795112933739450
191	2·28103336724772753763504359827061031849573613417824
192	2·28330122870354960857746127160207346980926815296335
193	2·28555730900777376059723864635331082109792160194605
194	2·28780172993022604699810070639007047619403146610098
195	2·29003461136251801128779416647295071326166817111566
196	2·29225607135647605185191030663425844050352455557215
197	2·29446622616159292737174431769715501751206467200453
198	2·29666519026153111055399467247774788687514980030995
199	2·29885307640970665010002178441980284149488877149827
200	2 30102999566398119521373889472449302676818988146211

N.	Logarithms to 50 Decimal Places.
201	2·30319605742048887144415953248118389014975494475931
202	2·30535136944662376948892707294743093998747923666856
203	2·30749603791321291804506302156189118473786681421112
204	2·30963016742589875626267558703243839274407600553996
205	2·31175385105575429929567295524368777276232375237158
206	2·31386722036915340038481008930473245115871337843814
207	2·31597034545691775345783291876142016809776876271749
208	2·31806333496276155006146073684030033790248871423550
209	2·32014628611105400228653344699995355965783152786095
210	2·32221929473391926800724416184775150268370126051866
211	2·32428245529769266508155812992314983375455169227144
212	2·32633586093875143606047008107624264623188216421915
213	2·32837960343873772338785733829054444447054876579109
214	2·33041377334919083604828013418592867289500677309613
215	2·33243845991560533119134925850472912204058500291798
216	2·33445375115093089752630039393882500790495623695841
217	2·33645973384852951037892035871105191578727395462815
218	2·33845649360460483041425220233836831341039440429009
219	2·34044411484011833836941480372542384365299711950235
220	2·34242268082220623596393886596751726847489207192856
221	2·34439227368511069774667505227066546083729656681212
222	2·34635297445063862931721748751873778395129143581912

It need scarcely be remarked, that the logarithm of any composite number whose factors, or the root of one or more of its factors, are less than 221, can be readily computed from this table. The logarithms thus computed may be had true to any number of decimal places to 49.

THE END.

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